Statistica & Applicazioni - Vol. VI, n. 2, 2008, pp. 181-192

ESTIMATION OF POWER FUNCTION DISTRIBUTION WITH APPLICATION TO ECOLOGICAL RELATIVE ABUNDANCE

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SUMMARY

In this paper we derive Bayesian and non-Bayesian estimators for the parameter of the power function distribution, and prediction intervals for the maximum of a future sample. We apply our approach to field data of plant species relative abundance, the abundance of a given species divided by the total abundance of all plant species in given a community, collected in a biodiversity project in central Europe.

Keywords: *Power function distribution, Bayes estimator, prior distribution, plant relative abundance.*

1. INTRODUCTION

Amongst the basic information needed when studying the ecological diversity of a given community, such as animals, plants, and insects, is the knowledge of the species relative abundance (SRA). The SRA is typically computed by dividing the abundance of a particular species by the total abundance of all other species found in the same community.

In addition to the knowledge gained from investigating the ecological diversity of biological communities, estimating SRA in conservation biology is of considerable significance. For example, resolving the relative abundances of a variety of species in a given community helps us to determine which species are common and which are rare within and among communities (Soule, 1986). Several theoretical and statistical models have been developed and used to examine the SRA (reviewed in McGill et al., 2007). In this paper, we use the power function distribution to model the relative abundances of species collected from various sites in Central Germany in the framework of a broad biodiversity project (Kahmen et al., 2005).

A random variable X is said to have a power function distribution with parameter θ, denoted by $X \sim PF(\theta)$, if it has the density function

$$f(x;\theta) = \begin{cases} \dfrac{\theta}{1-\theta} x^{\frac{2\theta-1}{1-\theta}}, & 0 < x < 1,\ 0 < \theta < 1; \\ 0 & o.w. \end{cases}$$

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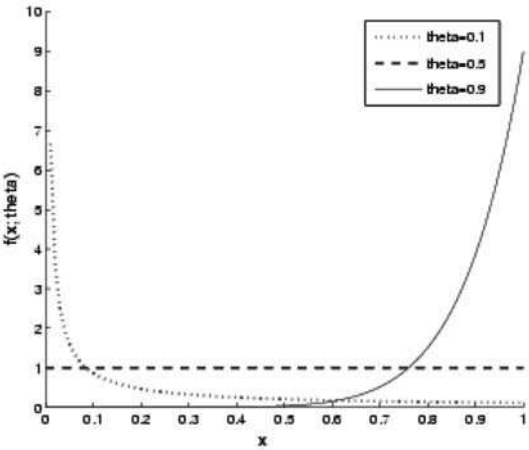

FIGURE 1. - *The pdf of $PF(\theta)$ for different values of θ*

Meniconi and Barry (1996) used the power function distribution as a model for the failure rates data obtained from two types of destructive tests. Rider (1964) derived distributions for the products and ratios of the order statistics from $PF(\theta)$. The moments of its order statistics are derived by Malik (1967). Lwin (1972) and Arnold and Press (1983) discussed a Bayesian estimation for the scale parameter of Pareto distribution using power function prior. Ali et al. (2000) calculated a MVUE for the right tail probability for $PF(\theta)$. Masoom et al. (2005) derived the distribution of $X/(X+Y)$ when X and Y are independent. Findings of these independent reports clearly indicate that the power function distribution has some important applications in reliability analyses and is less complex compared to other distributions such as Weibull and lognormal. Using the power function distribution, it could also be useful to model numerous environmental data, such species relative abundances. Unfortunately, we are unaware of any studies that have dealt with Bayesian and non-Bayesian estimation as well as Bayesian and non-Bayesian prediction when the underlying model is the power function distribution. In this study, we derive Bayesian and non-Bayesian estimators for the parameter the power function distribution, and derive prediction intervals for the maximum of a future sample. Also, we apply our approach to field data of plant species relative abundance collected in a biodiversity project in central Europe (see Perner et al., 2005 for details).

2. NON-BAYESIAN ESTIMATION

It is easy to see $EX = \theta$, $Var(X) = \dfrac{\theta(1-\theta)^2}{2-\theta}$ and the CDF of X is

$F(x;\theta) = 0$ for $x < 0$, $F(x;\theta) = x^{\frac{\theta}{1-\theta}}$, for $0 \leq x < 1$ and $F(x;\theta) = 1$, for $x \geq 1$.

The Fisher's information number about θ contained in a single observation X is

$$I_X(\theta) = -E\left\{\frac{\partial^2 \log f(X;\theta)}{\partial\theta^2}\right\},$$
$$= -E\left\{-\frac{1}{\theta^2} + \frac{1}{(1-\theta)^2} + \frac{2\log X}{(1-\theta)^3}\right\},$$
$$= \frac{1}{\theta^2(1-\theta)^2}.$$

Moreover, the i^{th} order statistic of a random sample of size n form $PF(\theta)$ is

$$g_i(y;\theta) = \frac{n}{(i-1)(n-i)}\frac{\theta}{1-\theta}y^{\frac{(i+1)\theta-1}{1-\theta}}\left(1-y^{\frac{\theta}{1-\theta}}\right)^{n-i}, \text{ for } 0 < y < 1;$$

and $g_i(y;\theta) = 0$, otherwise. For $i = n$, the density of the maximum is

$$g_n(y;\theta) = \frac{n\theta}{1-\theta}y^{\frac{(n+1)\theta-1}{1-\theta}}, \text{ for } 0 < y < 1;$$

and $g_n(y;\theta) = 0$, otherwise. If $Y = -\log(X)$, then $Y \sim \exp\left(\frac{1-\theta}{\theta}\right)$.

It is apparent that the power function distribution belongs to the regular exponential class and the statistic $T = -\log(\prod_{i=1}^{n} X_i) = -\sum_{i=1}^{n}\log(X_i)$ is a complete sufficient statistic for θ , where $X_1,\ldots,X_n$is a random sample from$PF(\theta)$. Moreover, $T \sim Gam\left(n, \frac{1-\theta}{\theta}\right)$. So 1+$T/n$ is a MVUE for $\frac{1}{\theta}$. A MVUE for θ can be obtained by solving the functional equation $E\{h(T)\} = \theta$ for h, i.e., we need to find the function h such that

$$\int_0^\infty \frac{h(t)t^{n-1}\exp\left(-\frac{\theta t}{1-\theta}\right)}{\Gamma(n)\theta^{-n}(1-\theta)^n}dt = \theta \qquad \text{for every } \theta.$$

By using the substitution $\beta = \theta/(1-\theta)$ to rearrange the last equation, we get: $\int_0^\infty h(t)t^{n-1}\exp(-\beta t)dt = \frac{\Gamma(n)}{\beta^{n-1}(1+\beta)}$. This implies that $h(t)t^{n-1}$ is the inverse Laplace transform of $\frac{\Gamma(n)}{\beta^{n-1}(1+\beta)}$, which can be obtained via complex integration. The inverse Laplace transform of $\frac{\Gamma(n)}{\beta^{n-1}(1+\beta)}$ is given by

$$h(t)t^{n-1} = (-1)^n (n-1)\exp(-t)\{\Gamma(n-1,-t) - \Gamma(n-1)\},$$

where $\Gamma(.,.)$ is the incomplete Gamma function. So the MVUE for θ is given by

$$h(T) = \frac{(-1)^n (n-1)\exp(-T)\{\Gamma(n-1,-T) - \Gamma(n-1)\}}{T^{n-1}}.$$

The method of moment (MOM) estimator for θ is $\hat{\theta}_{MOM} = \overline{X}$, the sample mean of $X_1, \ldots, X_n$. Moreover, $Var(\hat{\theta}_{MOM}) = \dfrac{\theta(1-\theta)^2}{n(2-\theta)}$. The maximum likelihood estimator (MLE) of θ is value which maximizes the likelihood function $L(\theta; t)$, where

$$L(\theta; t) = \theta^n (1-\theta)^{-n} \exp\left(-\frac{\theta t}{1-\theta}\right) \exp(t).$$

Simple calculations show that the MLE of θ is $\hat{\theta}_{MLE} = \dfrac{1}{1+T/n}$. The variance of the $\hat{\theta}_{MLE}$ has no closed form. The quantity $\dfrac{2\theta T}{1-\theta} \sim X^2_{2n}$ is a pivot fot θ.

3. NON-BAYESIAN PREDICTION INTERVALS

Let $X_1, \ldots, X_n$ be an experimental sample from $PF(\theta)$ and $Y_1, \ldots, Y_m$ be a future (unobserved) sample from $PF(\theta)$. Let $S_1 = S_1(X_1, \cdots, X_n)$ and $S_2 = S_2(Y_1, \cdots, Y_m)$ be two statistics in $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$, respectively. To find a prediction interval for the statistic S_2 we first find another statistic $S_3 = S_3(S_1, S_2)$ such that the distribution of S_3 does not depend on any parameter. Let a and b be the $(\alpha/2)100$ and the $(1-\alpha/2)100$ percentiles of the distribution of S_3. A $(1-\alpha)100\%$ prediction interval for S_2 is given by $[l,\ u]$, where l and u are the solutions of the equations $a = S_3(S_1, S_2)$ and $b = S_3(S_1, S_2)$. Now we provide the following two prediction intervals:

3.1 *Prediction of one future observation*

Let X_{n+1} be a future observation from $PF(\theta)$. Since $\dfrac{2T\theta}{1-\theta} \sim \chi^2_{2n}$ and $\dfrac{-2\theta \log X_{n+1}}{1-\theta} \sim \chi^2_2$, then

$$\frac{2\theta T/(2n(1-\theta))}{-2\theta \log X_{n+1}/(2(1-\theta))} = \frac{T}{-n \log X_{n+1}} \sim F_{2n,2},$$

where $F_{2n,2}$stands for the Fisher's random variable with $2n$ and 2 degrees of freedom. Let $l = F_{2n,2}(\alpha/2)$ and $u = F_{2n,2}(1-\alpha/2)$ be the $(\alpha/2)100$ and $(1-\alpha/2)100$ percentiles of the $F_{2n,2}$ distribution. This implies that

$$P\left\{l \leq \frac{T}{-n\log X_{n+1}} \leq u\right\} = 1-\alpha.$$

Solving the inequality $l \leq \dfrac{T}{-n\log X_{n+1}} \leq u$ for X_{n+1} yields the following $(1-\alpha)100\%$ prediction interval for X_{n+1}

$$\exp\left(-\frac{T}{nF_{2n,2}(\alpha/2)}\right) \leq X_{n+1} \leq \exp\left(-\frac{T}{nF_{2n,2}(1-\alpha/2)}\right). \quad (3.1)$$

3.2 *Prediction of maximum*

Since the density of $Y = \max(Y_1, \cdots, Y_m)$ is

$$g_n(y;\theta) = \frac{n\theta}{1-\theta} y^{\frac{(n+1)\theta-1}{1-\theta}}, \text{ for } 0 < y < 1,$$

then $V = -\log Y \sim Gam\left(1, \dfrac{1-\theta}{m\theta}\right)$. So $\dfrac{2m\theta V}{2(1-\theta)} \sim \chi^2_2$. The statistic $\dfrac{T}{nmV} = \dfrac{2\theta T/(2n(1-\theta))}{2m\theta V/(2(1-\theta))} \sim F_{2n,2}$. Following a similar argument as in part a we reach to the following $(1-\alpha)100\%$ interval for Y

$$\exp\left(-\frac{T}{mnF_{2n,2}(\alpha/2)}\right) \leq Y \leq \exp\left(-\frac{T}{mnF_{2n,2}(1-\alpha/2)}\right). \quad (3.2)$$

4. BAYESIAN ESTIMATION

Let $X_1, \ldots, X_n$ be a random sample from $PF(\theta)$. Assume that θ has the prior distribution $\pi(\theta)$. We are interested in making estimation and prediction when $\pi(\theta)$ is a non-informative prior or U(0, 1), a standard uniform distribution. Since T is sufficient statistic for θ, it suffices to write the likelihood function in term of t, i.e., the likelihood function is

$$L(\theta;t) = \theta^n(1-\theta)^{-n}\exp\left(-\frac{2\theta-1}{1-\theta}t\right) = \theta^n(1-\theta)^{-n}\exp\left(-\frac{\theta}{1-\theta}t\right)\exp(t).$$

4.1 $\pi(\theta)$ *is* $U(0,1)$

In this case the posterior distribution of θ given $T = t$ is

$$\pi(\theta|t) = \frac{L(\theta;t)\pi(\theta)}{\int_0^1 L(\theta;t)\pi(\theta)d\theta},$$

$$= \frac{\theta^n(1-\theta)^{-n}\exp\left(-\frac{\theta t}{1-\theta}\right)}{\int_0^1 \theta^n(1-\theta)^{-n}\exp\left(-\frac{\theta t}{1-\theta}\right)d\theta}, \qquad 0 < \theta < 1.$$

Using the Transformation
$v = \theta/(1-\theta) \Rightarrow \theta = v/(1-v)$ and $d\theta = dv/(1+v)^2$, we get

$$\pi(\theta|t) = \frac{\exp\left(-\frac{\theta}{1-\theta}t\right)\left(\frac{\theta}{1-\theta}\right)^n}{\int_0^\infty v^n(1+v)^{-2}\exp(-vt)dv}$$

Under the squared error loss function, $l(\theta, \delta) = (\delta - \theta)^2$, the Bayes estimator of θ, is the one which minimizes the posterior expected loss, i.e., the Bayes estimator is

$$\hat{\theta}_{1b}^U = E(\theta|T) = \frac{\int_0^1 \theta^{n+1}(1-\theta)^{-n}\exp\left(-\frac{\theta}{1-\theta}t\right)d\theta}{\int_0^1 \theta^n(1-\theta)^{-n}\exp\left(-\frac{\theta}{1-\theta}t\right)d\theta},$$

$$= \frac{\int_0^\infty v^{n+1}(1+v)^{-3}\exp(-vt)dv}{\int_0^\infty v^n(1+v)^{-2}\exp(-vt)dv}.$$

and the Bayes estimator of θ^2 is

$$\hat{\theta}_{2b}^U = E(\theta^2|t) = \frac{\int_0^1 \theta^{n+2}(1-\theta)^{-n}\exp\left(-\frac{\theta}{1-\theta}t\right)d\theta}{\int_0^1 \theta^n(1-\theta)^{-n}\exp\left(-\frac{\theta}{1-\theta}t\right)d\theta},$$

$$= \frac{\int_0^\infty v^{n+2}(1+v)^{-4}\exp(-vt)dv}{\int_0^\infty v^n(1+v)^{-2}\exp(-vt)dv}.$$

We use the posterior variance of $\hat{\theta}_{1b}^{U}$ as a measure of uncertainty for the estimator $\hat{\theta}_{1b}^{U}$. The posterior variance is $Var^{U}(\hat{\theta}_{1b}^{U}) = E(\theta^2|t) - \hat{\theta}_{1b}^{U}2$.

A $(1-\alpha)100\%$ High Posterior Density Credible Region (HPDCR) for θ is the set $\Omega^U = \{\theta \in [0,1]: \quad \pi(\theta|t) \geq c\}$ for which $P(\theta \in \Omega^U|t) = 1-\alpha$. To obtain this set, numerical methods are needed.

4.2 $\pi(\theta)$ *is the Jeffery's prior*

The Jeffery's non-informative prior is given by $\pi(\theta) = \sqrt{I_X(\theta)} = \theta^{-1}(1-\theta)^{-1}$. Using this prior, the posterior distribution is

$$\pi(\theta|t) = \frac{\theta^{n-1}(1-\theta)^{-n-1}\exp\left(-\frac{\theta t}{1-\theta}\right)}{\int_0^1 \theta^{n-1}(1-\theta)^{-n-1}\exp\left(-\frac{\theta t}{1-\theta}\right)d\theta},$$

$$= \frac{t^n}{\Gamma(n)}\theta^{n-1}(1-\theta)^{-n-1}\exp\left(-\frac{\theta t}{1-\theta}\right).$$

The Bayes estimators for θ and θ^2 are

$$\hat{\theta}_{1b}^{J} = \frac{t^n}{\Gamma(n)}\int_0^\infty v^n(1+v)^{-1}\exp(-vt)dv$$

and

$$\hat{\theta}_{2b}^{J} = \frac{t^n}{\Gamma(n)}\int_0^\infty v^{n+1}(1+v)^{-2}\exp(-vt)dv,$$

respectively.

5. BAYESIAN PREDICTION INTERVALS

In this section prediction intervals for one future observation and for the order statistic of a future sample is derived. The prediction is performed under two kinds of prior distributions, uniform distribution and Jeffery's non-informative prior.

5.1 *Prediction of future observation and maximum when* $\pi(\theta)$ *is* $U(0,1)$

Let X_{n+1} be a new observation from $PF(\theta)$. We are interested in predicting X_{n+1} based on the observed sample $x_1, \ldots, x_n$. Thus, the predictive density of X_{n+1} given $x_1, \ldots, x_n$ is

$$f(x_{n+1}|t) = \int_0^1 f(x_{n+1}|\theta)\pi(\theta|t)d\theta,$$

$$= \frac{\int_0^1 \theta^{n+1}(1-\theta)^{-n-1}\exp\left(-\frac{\theta t}{1-\theta} + \frac{(2\theta-1)}{1-\theta}\log x_{n+1}\right)d\theta}{\int_0^1 \theta^n(1-\theta)^{-n}\exp(-\frac{\theta t}{1-\theta})d\theta},$$

$$= \frac{1}{x_{n+1}} \frac{\int_0^\infty v^{n+1}(1+v)^{-2}\exp(-v(t-\log x_{n+1}))dv}{\int_0^\infty v^n(1+v)^{-2}\exp(-vt)dv}.$$

We use $E(X_{n+1}|t)$ to predict X_{n+1} with prediction error $Var(X_{n+1}|t)$.

A $(1-\alpha)100\%$ prediction interval for X_{n+1} is $[a,b]$ where a and b obtained are the solutions of

$$\int_0^a f(x_{n+1}|t)dx_{n+1} = \alpha/2 \quad \text{and} \quad \int_0^b f(x_{n+1}|t)dx_{n+1} = 1-\alpha/2, \tag{5.1}$$

respectively.

The predictive density of Y, the maximum of a future sample $Y_1,\ldots,Y_n$ from $PF(\theta)$, is

$$f(y|t) = \frac{n\int_0^1 \theta^{n+1}(1-\theta)^{-n-1}\exp\left(-\frac{\theta t}{1-\theta} + \frac{(n+1)\theta-1}{1-\theta}\log y\right)d\theta}{\int_0^1 \theta^n(1-\theta)^{-n}\exp\left(-\frac{\theta t}{1-\theta}\right)d\theta},$$

$$= \frac{n\int_0^\infty v^{n+1}(1+v)^{-2}\exp(-vt+\{(n+1)v-(v+1)\}\log y)dv}{\int_0^\infty v^n(1+v)^{-2}\exp(-vt)dv},$$

$$= \frac{n}{y}\frac{\int_0^\infty v^{n+1}(1+v)^{-2}\exp(-v(t-n\log y))dv}{\int_0^\infty v^n(1+v)^{-2}\exp(-vt)dv}.$$

5.2 *Prediction of future observation and maximum for Jeffery's prior*

The predictive density of a new observation X_{n+1} given t is obtained by

$$f(x_{n+1}|t) = \frac{\int_0^1 \theta^n (1-\theta)^{-n-2} \exp\left(-\frac{\theta t}{1-\theta} + \frac{2\theta - 1}{1-\theta}\log x_{n+1}\right) d\theta}{\Gamma(n) t^{-n}},$$

$$= \frac{nt^n}{x_{n+1}(t - \log x_{n+1})^{n+1}}; \quad \text{for } 0 < x_{n+1} < 1 \tag{5.2}$$

The prediction mean and variance as well as the prediction interval for X_{n+1} can be obtained similarly as in the uniform case. To predict Y, the maximum of a future sample $Y_1, \ldots, Y_n$ we need the following predictive density of Ygiven t:

$$f(y|t) = \frac{n\int_0^1 \theta^n (1-\theta)^{-n-2} \exp\left(-\frac{\theta t}{1-\theta} + \frac{(n+1)\theta - 1}{1-\theta}\log y\right) d\theta}{\int_0^1 \theta^{n-1} (1-\theta)^{-n-1} \exp\left(-\frac{\theta t}{1-\theta}\right) d\theta},$$

$$= \frac{n\int_0^\infty v^n \exp(-vt + \{(n+1)v - (v+1)\}\log y) dv}{\int_0^\infty v^{n-1} \exp(-vt) dv},$$

$$= \frac{n^2 t^n}{y(t - n\log y)^{n+1}}, \qquad 0 < y < 1,$$

and $f(y|t) = 0$, otherwise. Solving the equations

$$\int_0^l \frac{n^2 t^n}{y(t - n\log y)^{n+1}} dy = \alpha/2 \quad \text{and} \quad \int_0^u \frac{n^2 t^n}{y(t - n\log y)^{n+1}} dy = 1 - \alpha/2$$

for $l$and u we obtain the following $(1-\alpha)100\%$ prediction interval for Y

$$\left[\exp\left(\frac{T}{n}(1 - (\alpha/2)^{-1/n}\right), \quad \exp\left(\frac{T}{n}(1 - (1-\alpha/2)^{-1/n}\right)\right].$$

6. APPLICATION TO REAL DATA

The data used in the present model is based on a biodiversity project carried out in the Thueringer Schiefergebirge/Frankenwald, a plateau-like mountain range at the Thuringian/Bavarian border in central Germany with a maximum height of 870 m

(Kahmen et al., 2005). Average annual temperature varies between 68^0 C and 78^0 C and average annual precipitation varies between 950 and 1099 mm, with a slight increase in June and July (Perner et al., 2005). The bedrock material in the investigated area produces a carbonate and nutrient poor soil. This work is based on data gathered from 71 grasslands communities located between 11.018^0 and $11.638\ ^0$ eastern longitudes and between 50.358^0 and 50.578^0 northern latitudes, and with an area of about one hectare. For our model, four different plant species were chosen, namely *Dactylis glomerata*, *Taraxacum officinale* agg., *Trifolium repens*, and *Veronica chamaedrys*. The species sample was collected from 19 sites that are almost comparable with regard to elevation, edaphic and climatic conditions. Further detailed description of the studied communities can be found in Kahmen et al. (2005).

Here, we provide data for *Dactylis glomerata* as a representative species from the 19 sites:

0.371	0.310	0.146	0.028	0.062	0.023	0.059	0.079	0.015	0.011
0.019	0.0001	0.038	0.025	0.044	0.063	0.158	0.002	0.001.	

Figure 2 shows the frequency histogram for these data as well as the power function density with $\theta = 0.2152$ computed from the data. We used the Kolmogrov-Smirnov test for testing the hypothesis that the above data set follows $PF(0.2152)$. The test produced a p-value of 1, which means that the data could be fitted by $PF(0.2152)$. Using equation (5.2) we predicted a new observation from this distribution. We found that the predicted value of X_{20} is 0.213335 with prediction error measures by 0.105539, the variance of the distribution given by this equation.

Using equation (5.1), the predicted value of X_{20} is 0.21245 with prediction variance 0.0416. Table 1 shows the values of the Bayes and non Bayes estimators together with their standard errors calculated from the above data, while table 2 presents Bayes and non Bayes 95% prediction intervals for a new site. From these results, we see say that the Bayesian estimators are better than the classical estimators since they have smaller standard errors. In fact, two non-informative priors are used here, the Jeffrey's prior and the uniform prior. The uniform prior is slightly more informative than the Jeffrey's prior. The results in table 1 and table 2 agree with this interpretation. So using more informative priors about the parameter θ, we can get better estimates and prediction intervals. produce slightly better estimator than the classical ones, which is an expecetd

TABLE 1. - *Values of Bayes and non Bayes estimators and their standard errors*

Estimator	*Estimate*	*Standard Error*
$\hat{\theta}_{MOM}$	0.07653	0.04226
$h(T)$	0.20813	0.03802
$\hat{\theta}_{MLE}$	0.21832	0.03815
$\hat{\theta}^{U}_{1b}$	0.21522	0.03302
$\hat{\theta}^{J}_{1b}$	0.21334	0.03601

TABLE 2. - *Bayes and non Bayes prediction intervals*

Interval Using Equation	*95% Prediction Interval*
3.1	[6.5400E-7, 0.9154]
3.2	[4.7000E-7, 0.9161]
5.1	[0.5130 E0, 0.9817]
5.2	[3.56490E-7, 0.911]

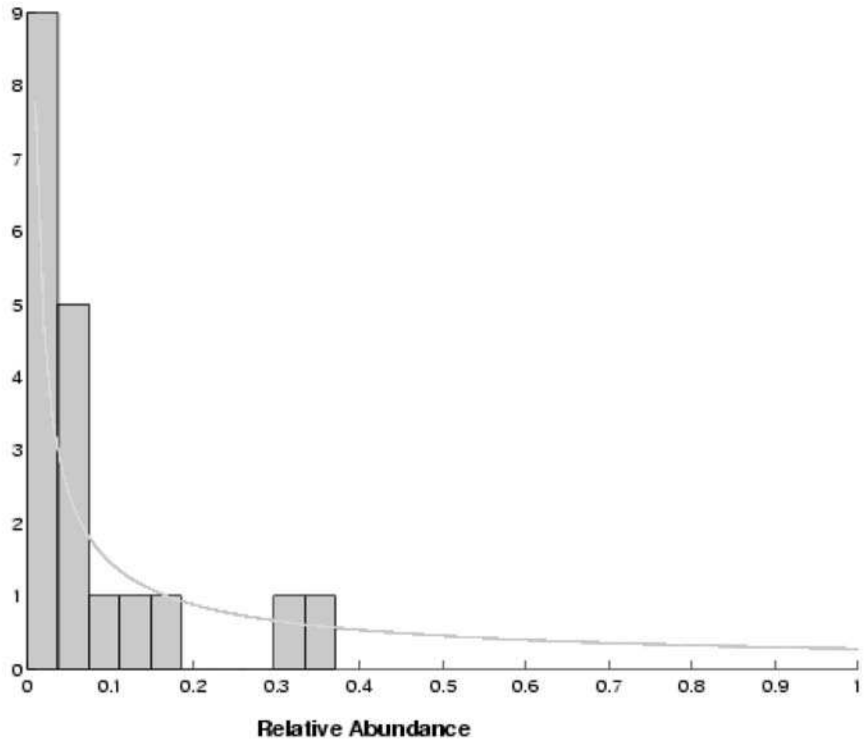

FIGURE 2. - *Theoretical density of power function distribution with the frequency histogram of the 19 sites relative abundances for* Dactylis glomerata

7. CONCLUSIONS

In this paper we derived Bayesian and non-Bayesian estimators for the mean of the power function distribution. We also derived Bayesian and non-Bayesian prediction intervals for a future observation and for the maximum of a future sample. Some results have closed forms while the others require numerical calculations. We also applied our findings to real data. The results show that the Bayesian statistics gives better results than the classical statistics.

RIASSUNTO

L'abbondanza relativa delle specie (SRA) è un'informazione fondamentale nello studio della diversità biologica di una comunità. In letteratura sono proposti diversi modelli statistici per la sua rappresentazione, nel presente contributo viene considerata la power function distribution. Dopo aver dedotto gli stimatori bayesiani e non bayesiani del parametro della distribuzione stessa, sono proposte stime bayesiane e non degli intervalli di predizione dell'osservazione a valore massimo in un campione futuro. Questi risultati sono applicati ad un data set relativo a specie di piante nell'ambito di un progetto per lo studio della biodiversità nell'Europa centrale.

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