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COMPLEMENTARITY MODELS FOR RESTRUCTURED ELECTRICITY MARKETS UNDER ENVIRONMENTAL REGULATIONS

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SUMMARY

Complementarity problems are recognized to be a general computational method for solving economic equilibrium models. There exist various problems describing the energy markets that rely on the complementarity models since they allow to analyze the interactions among different market players. Complementarity models generalize linear and non-linear problems because the Karush-Kuhn-Tucker optimality conditions are one particular instance of a complementarity problem. Moreover, the class of complementarity models is appropriate for modeling spatial price equilibrium, perfect and imperfect completion models, such as Cournot-Nash games, and other many models where both primal and dual variables can be constrained together. The first part of this paper provides a motivation and a description of complementarity models. In the second part, we investigate a capacity expansion problem applied to the restructured Italian electricity market that is currently subject to the European Union Emissions Trading System (EU-ETS). In accordance with the Kyoto Protocol, the EU-ETS aims to reduce greenhouse gas emissions from human activities provoking climate changes. This scheme is now subdivided into three phases and it is based on a cap and trade system that defines the maximum amount of CO_2 *that can be emitted in each compliance period. Our analysis shows that investments in renewables are mainly conditioned to incentive policies. The solution of the developed model is found by exploiting the mixed complementarity theoretical framework. The model is implemented in GAMS using the PATH solver.*

Keywords: *Complementarity Models, EU-ETS, Investments, Italian Electricity Market.*

1. INTRODUCTION

The restructuring of the electricity system has deeply changed the organization of this market around the world.The European Commission liberalized the electricity market though Directive 96/92/EC. This Directive imposed the unbundling of generation, transmission and distribution that, since then, were vertically integrated and controlled by a sole entity, usually a power company controlled by the national Government. The aim of this Directive was to improve efficiency and avoid abuses of dominant position, especially in setting power prices.

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In Italy, the disposals of Directive 96/92/EC were acknowledged by Bersani decree in 1999, but only in 2004 the Power Exchange GME (''Gestore del Mercato Elettrico'') became operative on Italian energy market (IPEX). With the Bersani decree, the old monopolist Enel had to disinvest 15 GW of its production capacity in order to reduce its market share. Enel currently contributes for the 26.4% to the Italian power production compared to the 43.9% of 2004[1]. However, the Italian market cannot be yet considered fully competitive as highlighted by Floro (2009).

Since 2005, the energy sector is involved in the European Union Emissions Trading System[2] (EU-ETS). The EU-ETS is an environmental policy developed in theframework of the Kyoto Protocol that aims at reducing the European CO_2 emissions generated by carbon-intensive installations of the energy and industrial sectors. Such a goal is achieved through the implementation of a cap-and-trade system that imposes a CO_2 emission ceiling on all covered installations and creates a market that prices CO_2 and where ETS participants can exchange their emission permits. The EU-ETS was initially subdivided into two phases as indicated by Directive 2003/87/EC. The first phase (2005-2007), the so-called ''learning by doing phase'', was introduced to test the functioning of the EU-ETS system. Its implementation led to some economic distortions mainly due to the grandfathering of the emission allowances (Neuhoff *et al.,* 2006a, 2006b; Reinaud, 2003, 2005) and to the consequent raise of ''windfall profits'' for the power sector (Sijm *et al.,* 2006). Compared to the energy intensive industries involved in the EU-ETS, generators are able to pass through a high proportion of their carbon costs in electricity prices despite the fact that almost all CO_2 permits, needed to cover their emissions, are freely distributed. The result is twofold: the EU-ETS causes both an increase of electricity prices and an unintentional raise of generators' profits. These two issues have been extensively discussed in literature and many studies confirm this outcome (see, for instance, Chen *et al.*, 2008; Kara *et al.*, 2008; Linares *et al.*, 2008 and Lise *et al.*, 2010; Oggioni and Smeers, 2009).

In order to remedy to this situation, Directive 2009/29/EC, regulating the third EU-ETS phase (2013-2020),has imposed a full auctioning system for the allocation of emission permits destined to the energy sector.For the industrial sectors, it foresees a progressive adoption of an auctioning system starting from a proportion of the 20% in 2013 and reaching a 70% level in 2020[3] .Moreover, the revised EU-ETS will cover more industries and types of greenhouse gases and will encourage the development of renewables.

In this paper, we investigate the economic impacts of the EU-ETS on the Italian electricity market. In particular, taking into account the current organization of the

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[1] See Autorità per l'Energia Elettrica e il Gas (AEEG), Relazione annuale sullo stato dei servizi e sull'attività svolta, 2012. Available at http://www.autorita.energia.it/it/relaz_ann/12/12.htm.

[2] http://ec.europa.eu/clima/policies/ets/index_en.htm.

[3] The Article 10 ter of the Directive 2009/29/EC states that all industrial sectors that are exposed to the risk of carbon leakage will continue to receive free permits. See http://ec.europa.eu/clima/policies/ets/leakage/index_en.htm.

Italian electricity market, we formulate a capacity expansion model, where generators are Cournot players that operate in different zones linked by inter-connectors with limited transfer capacity. Imperfect competition models are often used to study electricity markets (see, for instance, Murphy and Smeers, 2005; Pineau and Murto, 2003; Ventosa *et al.*, 2002). An oligopolistic market can be described either by Nash-Cournot or Bertrand or Supply Function Equilibrium models. Supply Function Equilibrium (see Anderson and Hu, 2008; Willems *et al.*, 2009) and Cournot Equilibrium (for overviews see Tirole, 1988; Vives, 1999 and for review see Ventosa *et al.*, 2005) are the most applied models to electricity markets. Moreover, different mathematical approaches can be used to find the equilibrium of imperfectly competitive markets. Pineau and Murto (2003) solve their dynamic stochastic oligopoly model using both all players' first order optimality conditions (the Karush-Kuhn-Tucker conditions) and a variational inequality approach (see Facchinei and Pang, 2003; Nagurney, 1999 for more details). In particular, the Karush-Kuhn-Tucker optimality conditions involve all market players' optimization problems and define a mixed complementarity problem that is used to determine the market equilibrium (see, for instance, Chen and Hobbs, 2005; Hobbs, 2001; Hobbs and Helman, 2004; Gabriel *et al.*, 2013; García-Bertrand and Conejo, 2006; Schweppe *et al.*, 1988). Depending on the problem at hand, imperfect competition can be also described through a Mathematical Problem with Equilibrium Constraints (MPEC) that identifies a Stackelberg game where a firm plays the role of leader anticipating the reactions of the follower firms (see, for instance, Chen *et al.*, 2006; Ventosa *et al.*, 2002).

The main contribution and the originality of our paper consist in the adoption of a mixed complementarity problem to study the effects of the EU-ETS regulation on the Italian electricity market.

The organization of this paper is as follows. Section 2 describes the mathematical background with a particular focus on complementarity problems. Section 3 illustrates generators and Market Operator's optimization and complementarity problems. Results are discussed in Section 4 and, finally, Section 5 reports the conclusions.

2. MATHEMATICAL BACKGROUND: COMPLEMENTARITY PROBLEMS

Complementarity problems are recognized to be a general computational method for solving economic equilibrium models. A reason why complementarity problems are so important is that the concept of complementarity is strictly related to the notion of system equilibrium, as well as necessary optimality condition for mathematical programming problems. The importance of the theory as well as the applications has been well documented in the literature (see Facchinei and Pang, 2003; Ferris and Pang, 1997; Gabriel *et al.*, 2013; Nagurney, 1999).

Complementarity is also central to all constrained optimization problems. The well-known complementary slackness property in linear programming defines the fundamental role of complementarity in optimization. This property also applies in nonlinear programs and variational inequalities.The classical complementarity problem (CP), defined by a nonlinear function $F : K \to \mathbb{R}^n$, given a cone $K \subseteq \mathbb{R}^n$, is to find a vector $x \in \mathbb{R}^n$ such that:

$$K \ni x \perp F(x) \in K^* \tag{1}$$

where $K^* = \{d \in \mathbb{R}^n | v^T d \geq 0, \ \forall v \in K\}$ is the dual cone of K. We use the notation ‘‘$\perp$’’ to signify that, in addition to the state conditions $x \in K$ and $F(x) \in K$, the equation $x^T F(x) = 0$ also holds.

When K is the nonnegative orthant of $\mathbb{R}^n$, the complementarity problem is called nonlinear complementarity problem and is denoted by NCP. But the dual cone of the nonnegative orthant is the nonnegative orthant itself, so the nonlinear complementarity problem, given a mapping $F : \mathbb{R}^n_+ \to \mathbb{R}^n$, is the problem of finding a vector $x \in \mathbb{R}^n$ such that:

$$0 \leq x \perp F(x) \geq 0. \tag{2}$$

We recall that condition (2) can be alternatively defined as:

$$x \geq 0, \qquad F(x) \geq 0, \qquad F_i(x) \cdot x_i = 0 \quad \forall i = 1, ..., n. \tag{3}$$

Complementarity problems of this form arises as the Karush-Kuhn-Tucker (KKT) conditions of a constrained nonlinear program:

$$\begin{array}{ll} min_x & f(x) \\ \text{sub to} & g(x) \leq 0 \\ & x \geq 0 \end{array}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is a continuously differentiable real-valued function and $g : \mathbb{R}^n \to \mathbb{R}^p$ is a continuously differentiable vector-valued function.

The KKT conditions are necessary conditions that a solution to an optimization problem must satisfy. In fact, they can be used to analytically prove that a point is an optimum of a constrained problem. The KKT conditions are also sufficient conditions for optimality when the function f is convex and the feasible region is convex. The KKT conditions of the optimization problem presented above are:

$$0 \leq \nabla_x f(x) + \lambda^T \nabla_x g(x) \perp x \geq 0 \tag{4}$$

$$0 \geq g(x) \perp \lambda \geq 0 \tag{5}$$

where $\lambda \in \mathbb{R}^p$ is the Lagrange multiplier vectors and ∇_x is the gradient with respect to x. A generalization of the NCP is the mixed complementarity problem (MCP). Given $G : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2}_+ \to \mathbb{R}^{n_1}$ and $H : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2}_+ \to \mathbb{R}^{n_2}$, with $n_1 + n_2 = n$, the mixed complementarity problem is the problem of finding a pair of vectors (u, v) belonging to $\mathbb{R}^{n_1} \times \mathbb{R}^{n_2}$ such that:

$$G(u, v) = 0, \qquad u \ \text{ free} \tag{6}$$

$$0 \leq v \perp H(u, v) \geq 0. \tag{7}$$

The complementarity problem is also correlated to the variational inequality formulations of equilibrium problems. In fact, the variational inequality problem is closely related with many problems of Nonlinear Analysis, such as complementarity, fixed point and optimization problems.

Let K be a nonempty, closed and convex subset of the n-dimensional Euclidean space $\mathbb{R}^n$, $F : K \to \mathbb{R}^n$ a continuous mapping. The variational inequality problem (VI for short) is the problem of finding a point $x^* \in K$ such that

$$(x - x^*)^T F(x^*) \geq 0, \quad \forall\, x \in K. \tag{8}$$

The solution set of VI (8) is denoted by $SOL(K, F)$.

In particular, the variational inequality problem contains as a special case the complementarity problem. The relationship between them is provided by the following proposition.

Proposition 1
(see Nagurney, 1999) Let $K = \mathbb{R}^n_+$ be the nonnegative orthant, the variational inequality and the nonlinear complementarity problem have precisely the same solution, if any.

As shown by Hartmann and Stampacchia (1966), VI (8) has a solution if K is compact and F is continuous.

In general, VI can have more than one solution. We now recall one condition under which VI (8) has a unique solution, this result needs generalized monotonicity assumption.

Definition 1
Let K be a convex set in $\mathbb{R}^n$. A mapping $F : K \subseteq \mathbb{R}^n \to \mathbb{R}^n$ is said to be

- *monotone on K if $(F(x) - F(y))^T (x - y) \geq 0$, $\forall x, y \in K$;*
- *strictly monotone on K if $(F(x) - F(y))^T (x - y) > 0$, $\forall x, y \in K$ and $x \neq y$.*

Theorem 1
(see Harker and Pang, 1990) If $F(x)$ is strictly monotone then VI (8) has at most one solution.

In addition, we now recall the known monotonicity criteria for continuously differentiable mappings.

Theorem 2
(see Ortega and Rheinboldt, 1970) Let K be an open convex set in $\mathbb{R}^n$ and let $F : K \subseteq \mathbb{R}^n \to \mathbb{R}^n$ be continuously differentiable on K.

- *F is monotone on K if and only if ∇F is positive semidefinite on K;*
- *F is strictly monotone on K if ∇F is positive definite on K.*

It is possible to formulate a VI problem for a Nash equilibrium of several agents. In addition, a large number of economic models are formulated in terms of N-person noncooperative game. In the general setting, such a game consists of N players, each of whom has a certain cost function $\theta_i(x)$, where $x = (x^i : i = 1, ..., N)$, and a strategy set $K_i \subseteq \mathbb{R}^{n_i}$.

In a Nash equilibrium problem, each player minimizes his own costs assuming that the other players' strategies $\widetilde{x}^i = (x^j : j \neq i)$ are known. In other words, the problem of player i is to solve the cost minimization problem in the variable y^i, given the other players' strategies $\widetilde{x}^i = (x^j : j \neq i)$:

$$\begin{aligned} &\min \quad \theta_i(y^i, \widetilde{x}^i) \\ &\text{sub to} \;\; y^i \in K_i. \end{aligned} \tag{9}$$

The solution set of the problem is denoted by $S_i(\widetilde{x}^i)$.

Definition 2
A Nash Equilibrium is a tuple of strategies $x = (x^i : i = 1, ..., N)$ *such that for each i,* $x^i \in S_i(\widetilde{x}^i)$.

This problem can often be transformed into an equivalent variational inequality or complementarity problem if for each fixed $\widetilde{x}^i$ the $\theta_i(y^i, \widetilde{x}^i)$ function is convex and continuously differentiableJin y^i. The following proposition gives the relationship between the solution of VI and the solution of a Nash Equilibrium (see Facchinei and Pang, 2003).

Proposition 2
Let K_i *be a close subset of* $\mathbb{R}^{n_i}$. *Assume that for each fixed tuple* $\widetilde{x}^i$, *the function* $\theta_i(y^i, \widetilde{x}^i)$ *is convex and continuously differentiable in* y^i.
Then x is a Nash equilibrium if and only if $x \in SOL(K, F)$, *where*

$$K := K_1 \times K_2 \times ... \times K_N,$$

$$F(x) := (\nabla_{x^i}\theta_i(x)) \qquad i = 1, ..., N$$

and

$$\nabla_{x^i}\theta_i(x) = \left(\frac{\partial \theta_i(x)}{\partial x^{i1}}, ..., \frac{\partial \theta_i(x)}{\partial x^{iN}} \right).$$

3. THE MARKET MODEL

We develop an equilibrium problem that describes the restructured Italian electricity market. The equilibrium of this market defines the interactions between strategic generators and the Italian Market Operator. In particular, Market Operator maximizes consumers' willingness to pay by guaranteeing the respect of the transmission physical constraints and the satisfaction of the zonal energy balance. Electricity is produced by generators which compete à la Cournot. Their aim is to maximize their profits taking into account the decisions taken by their respective competitors. We also model an emission market limited to the energy sector. Market Operator and generators operate simultaneously in the power market. We first present the optimization problems solved by these two groups of operators and we then transform these problems in complementarity form.

3.1 *Notation*

We first list the sets, the parameters and the variables used in our models.

Sets

- $i \in I$: Zones;
- $t \in T$: Time segment, we consider time horizon $t = 1, ..., 24$ hours;
- $p \in P$: Set of technologies;
- $f \in F$: Generators.

Parameters

- $vc_{f,p,i}$: Hourly variable costs of new and existing plant oftechnology type p owned by generator f in zone i (€/MWh);
- $fc_{f,p,i}$: Hourly fixed costs of new plant of technology type $p$owned by generator f in zone i (€/MWh);
- $\overline{G}_{f,p,i}$: Total available capacity of technology plant$p$ owned by generator f in zone i (MW);
- e_p: Emission factor of technology p (ton/MWh);
- E: Total emission cap (ton);
- GE_f: Total amount of emission allowances grandfathered togenerators f in the period considered (ton);
- τ: 8760 (number of hours in one year);
- τ_t: 365 (number of days in one year);
- $a_{t,i}$: Intercept of consumers' affine demand functions at zone $i$in time segment t (€/MWh);
- $b_{t,i}$: Slope of consumers' affine demand functions at zone i intime segment t (€/MWh^2);
- $\overline{Flow}_{i,j}$: Flow transfer limit from zone $i$to zone $j \neq i$ where $j \in I$ (MW).

Variables

- $g_{t,f,p,i}$: Power produced by generators f in zone i usingexisting technology p in time segment t (MWh);
- $gn_{t,f,p,i}$: Power produced by generators f to zone i usingnew technology p in time segment t (MWh);
- $I_{f,p,i}$: Investments in new capacity of technology type p operated by generators f in zone i (MW);
- $s_{t,f,i}$: Power supplied by generator f to zone i in time segment t (MWh);

- $d_{t,i}$: Electricity consumption in zone i in time segment t (MWh);
- $p_{t,i}$: Zonal electricity price in time segment t (€/MWh);
- $P_{t,i}(d_{t,i})$: Willingness to pay in zone i and in time segment t (€€). This term can be explicitly defined as follows: $P_{t,i}(d_{t,i}) = a_{t,i} - b_{t,i} \cdot d_{t,i}$;
- $flow_{t,i,j}$: Power transferred from zone i to zone $j \neq i$ where $j \in I$ in time segment t.

3.2 *Generation expansion model*

We model a zonal market where generators compete as Cournot players on quantities. They produce energy by running existing or new plants in which they invest. In this model, generators build and operate both existing and new capacity in a single period where they also incur investment and operations costs. This standard static formulation assumes that new power plants are immediately available when built. The construction of these power plants is conducted by generators already operating in the market. Each (existing and new) plant is characterized by its own fixed[4], emission and fuel costs that influence their endogenously determined merit order. Generators make their strategical investments and production choices by taking into account the environmental opportunity costs due to the CO_2 regulation. Considering the assumption of Cournot competition among Italian generators, we assume that electricity prices are functions of quantities. More specifically, we define the zonal electricity prices $p_{t,i}$ as follows:

$$p_{t,i} = p_{t,i}\left(s_{t,-f,i} + s_{f,t,i} - \sum_j (flow_{t,i,j} - flow_{t,j,i})\right), \quad i \in I \quad t \in T \qquad (10)$$

This expression means that electricity prices $p_{t,i}$ are functions of the total amount of electricity sold on the market ($s_{t,-f,i} + s_{f,t,i}$) corrected by the amount of electricity exchanged ($\sum_j (flow_{t,i,j} - flow_{t,j,i})$)[5] between zones i and j. Note that the term $s_{t,-f,i}$ in (10) corresponds to the total amount of electricity sold in node i by all generators, with the exception of generator f. In mathematical terms, this can be also expressed as $s_{t,-f,i} = \sum_{f' \neq f} s_{t,f',i}$.

Considering this price formulation, each generator f maximizes its annual profits (11) taking into account the technological constraints (12)-(15). In particular, each generator gains by selling electricity at nodal prices $p_{t,i}$, as defined in (10), that depends on the total quantity sold and exchanged in each node i. Generators also face variable generation costs ($\sum_{p,i} vc_{f,p,i} \cdot (g_{t,f,p,i} + gn_{t,f,p,i}) \cdot \tau_t$), investment costs ($\sum_{p,i} fc_{f,p,i} \cdot I_{f,p,i} \cdot \tau$) and emission opportunity costs ($\varphi \cdot (GE_f - (\sum_{t,p,i} e_p \cdot (g_{t,f,p,i} + gn_{t,f,p,i}) \cdot \tau_t)))$ computed over a year. Note that the difference $[GE_f - (\sum_{t,p,i} e_p \cdot$

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[4] We only account for fixed costs of new generating units because we assume that existing plants are already fully amortized.

[5] Note that $flow_{t,i,j}$ stands for the amount of electricity exported from zone i to zone j ($i \neq j$ and $i,j \in I$); $flow_{t,i,j}$ indicates the import of zone i from zone j.

$\cdot(g_{t,f,p,i} + gn_{t,f,p,i}) \cdot \tau_t)]$ can be either positive or negative. It is positive if the amount of grandfathered allowances (GE_f) is higher than the CO$_2$ emissions generated $(\sum_{t,p,i} e_p \cdot (g_{t,f,p,i} + gn_{t,f,p,i}) \cdot \tau_t)$ when producing electricity; otherwise it is negative. In the first case, generators gain, while, in the second, incur in a cost. The allowance auctioning system foreseen by Directive 2009/29/EC can be simply modeled by setting GE_f equal to zero. Under this alternative assumption, generators have to pay for all their CO$_2$ emissions.

$$\begin{aligned} \text{Max} \quad & \left\{\sum_{t,i}\left[p_{t,i}\left(s_{t,-f,i} + s_{t,f,i} - \sum_j (flow_{t,i,j} - flow_{t,j,i})\right)\right] \cdot s_{t,f,i} \cdot \tau_t\right\} + \\ & -\left\{\sum_{t,p,i} vc_{f,p,i} \cdot (g_{t,f,p,i} + gn_{t,f,p,i}) \cdot \tau_t\right\} + \\ & +\left\{\varphi \cdot \left[GE_f - (\sum_{t,p,i} e_p \cdot (g_{t,f,p,i} + gn_{t,f,p,i}) \cdot \tau_t)\right]\right\} + \\ & -\sum_{p,i} fc_{f,p,i} \cdot I_{f,p,i} \cdot \tau \end{aligned} \tag{11}$$

s.t.

$$\sum_p g_{t,f,p,i} + \sum_p gn_{t,f,p,i} = s_{t,f,i} \qquad (\alpha_{t,f,i}) \qquad \forall t, f, i \tag{12}$$

$$\overline{G}_{f,p,i} - g_{t,f,p,i} \geq 0 \qquad (\beta_{t,f,p,i} \geq 0) \qquad \forall t, f, p, i \tag{13}$$

$$I_{f,p,i} - gn_{t,f,p,i} \geq 0 \qquad (\nu_{t,f,p,i} \geq 0) \qquad \forall t, f, p, i \tag{14}$$

$$g_{t,f,p,i}; gn_{t,f,p,i}; I_{t,f,p,i}; s_{t,f,i} \geq 0 \qquad \forall t, f, p, i \tag{15}$$

Considering the constraints, equation (12) defines a production balance between the total amount of electricity generated $(\sum_p g_{t,f,p,i} + \sum_p gn_{t,f,p,i})$ and sold $(s_{t,f,i})$ in zone i by generator f in each time segment t. Inequalities (13) and (14) impose generation capacity limits respectively on existing and new plants. Finally, conditions (15) define the non-negativity of the decision variables.

3.3 *Market Operator's model*

Market Operator maximizes the consumers' willingness to pay (16) taking into account a zonal energy balance (17), transmission (18), and non-negativity (19) constraints. Inequalities (18) limit the power flows exchanged among connected zones. The dual variable $\sigma_{t,i,j}$ associated to this constraint represents the transmission costs faced to transfer power from zones i to j. This variable assumes positive values when the line connecting zone i to j is congested.

$$Max_{d_{t,i}} \sum_t \left[\int_0^{d_{t,i}} P_{t,i}(\xi) d\xi \right] \cdot \tau_t \tag{16}$$

s.t

$$\sum_f s_{t,f,i} - \sum_j (flow_{t,i,j} - flow_{t,j,i}) - d_{t,i} = 0 \qquad (\psi_{t,i}) \qquad \forall t, i \tag{17}$$

$$0 \leq flow_{t,i,j} \leq \overline{Flow}_{i,j} \qquad (\sigma_{t,i,j}) \qquad \forall t, i, j \tag{18}$$

$$d_{t,i} \geq 0 \qquad \forall t, i \tag{19}$$

3.4 *Emission market*

CO_2 emissions are endogenously determined by the model. Taking into account the National Allowance Plans (NAPs) imposed by Directive 2003/87/EC, we introduce an emission constraint (20) that limits the carbon emissions generated by power plants. This constraint is associated to the dual variable φ, representing the allowance price. E indicates the annual CO_2 emission cap while $\sum_{t,f,p,i} e_p \cdot \cdot (g_{t,f,p,i} + gn_{t,f,p,i}) \cdot \tau_t$ are the annual emissions generated by electricity production.

$$E - \sum_{t,f,p,i} e_p \cdot \left((g_{t,f,p,i} + gn_{t,f,p,i}) \cdot \tau_t \right) \geq 0 \qquad (\varphi \geq 0) \tag{20}$$

3.5 *Complementarity problem*

In order to find the solution of our equilibrium problem, we formulate the generators and the Market Operator problems as complementarity problems. We also add the complementarity condition of the emission constraint (20). The result is a mixed complementarity problem that includes the KKT conditions of the generators and the Market Operator's problems. We finally get a mixed complementarity problem.More specifically, conditions (21)-(28) define the complementary formulation of the generators' optimization problem:

$$0 \leq vc_{f,p,i} + \varphi \cdot e_p - \alpha_{t,f,i} + \beta_{t,f,p,i} \perp g_{t,f,p,i} \geq 0 \qquad \forall t, f, p, i \tag{21}$$

$$0 \leq vc_{f,p,i} + \varphi \cdot e_p - \alpha_{t,f,i} + \nu_{t,f,p,i} \perp gn_{t,f,p,i} \geq 0 \qquad \forall t, f, p, i \tag{22}$$

$$0 \leq \frac{-\partial p_{t,i}}{\partial s_{t,f,i}} \cdot s_{t,f,i} - p_{t,i} + \alpha_{t,f,i} \perp s_{f,p,i} \geq 0 \qquad \forall f, p, i \tag{23}$$

$$0 \leq fc_{f,p,i} - \sum_t \frac{\tau_t}{\tau} \nu_{t,f,p,i} \perp I_{f,p,i} \geq 0 \qquad \forall f, p, i \tag{24}$$

$$\sum_p g_{t,f,p,i} + \sum_p gn_{t,f,p,i} - s_{t,f,i} = 0 \qquad (\alpha_{t,f,i}) \qquad \forall t, f, i \tag{25}$$

$$0 \leq \overline{G}_{f,p,i} - g_{t,f,p,i} \perp \beta_{t,f,p,i} \geq 0 \qquad \forall t,f,p,i \tag{26}$$

$$0 \leq I_{f,p,i} - gn_{t,f,p,i} \perp \nu_{t,f,p,i} \geq 0 \qquad \forall t,f,p,i \tag{27}$$

$$\beta_{t,f,h,i} - \overline{\xi}_{t,f,h,i} + \underline{\xi}_{t,f,h,i} = 0 \qquad (R_{t,f,h,i}) \qquad \forall t,f,h,i \tag{28}$$

Note that $p_{t,i}$ corresponds to the price defined in (10). The complementarity conditions associated to the Market Operator's problem are as follows:

$$0 \leq -a_{t,i} + b_{t,i} \cdot d_{t,i} + \psi_{t,i} \perp d_{t,i} \geq 0 \qquad \forall t,i \tag{29}$$

$$\sum_f s_{t,f,i} - \sum_j (flow_{t,i,j} - flow_{t,j,i}) - d_{t,i} = 0 \qquad (\psi_{t,i}) \qquad \forall t,i \tag{30}$$

$$0 \leq \overline{Flow}_{i,j} - flow_{t,i,j} \perp \sigma_{t,i,j} \geq 0 \qquad \forall t,i,j \tag{31}$$

$$0 \leq \psi_{t,i} - \psi_{t,j} + \sigma_{t,i,j} - \sigma_{t,j,i} \perp flow_{t,i,j} \geq 0 \qquad \forall t,i,j \tag{32}$$

Finally, (33) indicates the complementarity condition associated to the emission constraint (20).

$$0 \leq E - \sum_{t,f,p,i} e_p \cdot \left((g_{t,f,p,i} + gn_{t,f,p,i}) \cdot \tau_t \right) \perp \varphi \geq 0 \tag{33}$$

4. APPLICATION TO THE ITALIAN ELECTRICITY MARKET

4.1 *Case study*

We apply our model to the stylized Italian electricity market depicted on Figure 1. This market is subdivided into six geographical zones (North, Center-North, Center-South, South, Sicily and Sardinia) and five virtual poles with limited production (Monfalcone, Foggia, Brindisi, Rossano, Priolo). Some of the geographical zones group together several Italian regions. These are:

1. **North** which includes Valle d'Aosta, Piemonte, Liguria, Lombardia, Trentino-Alto Adige, Veneto, Friuli-Venezia Giulia and Emilia-Romagna. Note that Liguria, Veneto, Friuli Venezia Giulia and Emilia Romagna are coastal regions;
2. **Center-North** which assembles Toscana, Umbria and Marche;
3. **Center-South** which gathers Lazio, Abruzzo and Campania;
4. **South** which accounts for Molise, Puglia, Basilicata and Calabria.

Our analysis is based on 2009 data. Following the network representation provided by Terna, the Italian Transmission System Operator (TSO), we assume that each zone is linked to the others by the means of two connections with limited transfer capacities that depend on flow direction (see Terna, 2011). These transfer limits are listed in Table 35 and are used to define the parameter $\overline{Flow}_{i,j}$ in condition (18).

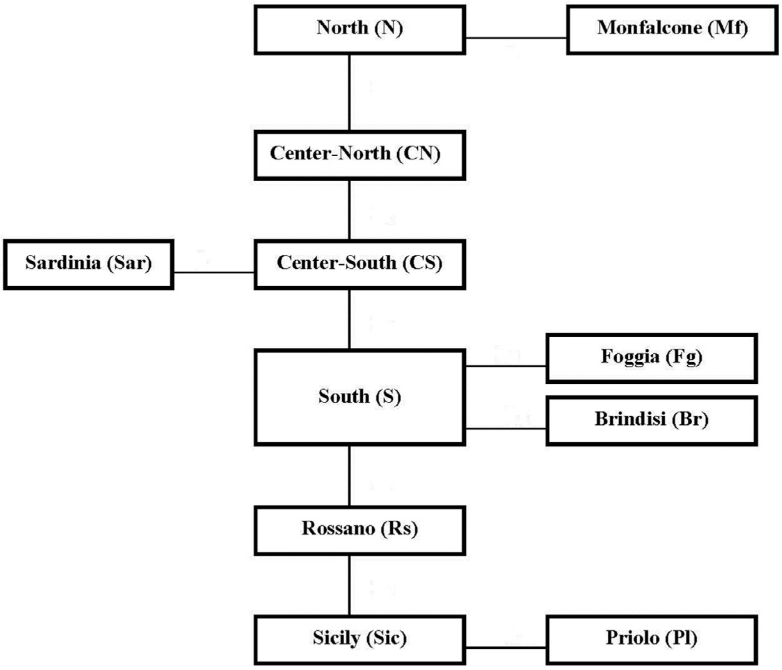

FIGURE 1. - *Italian network*

TABLE 1. - *Transfer limits of zonal interconnections*

MW	N	CN	CS	S	Sic	Sar	Mf	Fg	Br	Rs	Pl
N	0	3450	0	0	0	0	10000	0	0	0	0
CN	1700	0	1750	0	0	0	0	0	0	0	0
CS	0	2250	0	10000	0	420	0	0	0	0	0
S	0	0	3700	0	0	0	0	10000	10000	10000	0
Sic	0	0	0	0	0	0	0	0	0	275	10000
Sar	0	0	450	0	0	0	0	0	0	0	0
Mf	1030	0	0	0	0	0	0	0	0	0	0
Fg	0	0	0	1200	0	0	0	0	0	0	0
Br	0	0	0	5200	0	0	0	0	0	0	0
Rs	0	0	0	1613	150	0	0	0	0	0	0
Pl	0	0	0	0	315	0	0	0	0	0	0

A set of eight generators[6] produces electricity running wind, run-on-river, geothermal, photovoltaic, coal, CCGT, other gas and oil based power plants depend-

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[6] The considered companies are: Enel, Edison, Eni, Edipower, Eon, A2A, TirrenoPower and a fringe that collects all the remaining small power companies.

ing on their availability. Capacity data for all these technologies are taken from Terna[7] and from the annual reports of the main Italian power companies. Electricity is generated by existing and new power plants. We assume that generators can only invest in the geographical zones (namely North, Center-North, Center-South, South, Sicily and Sardinia). We do not impose any restrictions on investment choices except for hydro technologies since they are almost fully exploited in Italy. We recall that nuclear plants are not allowed. In order to simplify both the database and the interpretation of the results, we assume that old and new capacity have identical variable and emission costs. The models obviously allow one to change this assumption and to apply different efficiency rates to new plants. Doing so in this prototype study would however mix fundamental economic phenomena and sometimes arbitrary data differentiations and hence cloud the interpretation of the results. For this reason, we do not consider plant depreciation and maintenance costs.

The time horizon is a day subdivided into twenty-four hours. Empirical data referred to the last years showed that the variability of the Italian electricity consumption in the four seasons is low. The consumption in summer time is similar to that in winter because of the use of the air conditioning systems. For this reason, we decide to take a prototype day as reference time horizon. Electricity demand is modeled through an affine inverse demand function that depends on time and zones. Reference demand and prices used to compute demand parameters are taken from the Italian Market Operator website[8]. For all consumers, a 0.1 elasticity is assumed. We analyze the EU-ETS impacts on electricity prices, investments and generators' profits under different scenarios. A first group of scenarios describes the situation of the Italian electricity market in 2009, corresponding to the second EU-ETS phase, while a second group studies the new setting of the third EU-ETS phase in 2020. In all these scenarios, we assume that generators operate under the EU-ETS regime and can always invest in new capacity. Because renewable energy sources (RES) still needs to be incentivized today, we assume that generators receive subsidies in order to abate the fixed costs they face for investing in new wind and photovoltaic plants. Considering these assumptions, the scenarios analyzed are four:

1. ''2009-ETS'': this scenario reproduces the second EU-ETS phase. Investments in RES are not subsidized;
2. ''2009-ETS inc'': this scenario still analyzes the second EU-ETS phase, but investments in RES are subsidized with incentives;
3. ''2020-ETS'': this scenario studies the third EU-ETS phase. Investments in RES are not subsidized;
4. ''2020-ETS inc'': this scenario still analyzes the third EU-ETS phase, but investments in RES are subsidized with incentives.

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[7] See http://www.terna.it/default/Home/SISTEMA_ELETTRICO/statistiche/dati_statistici.aspx.

[8] See Gestore Mercato Elettrico (GME) website at http://www.mercatoelettrico.org/En/download/DownloadDati.aspx?val=MGP_PrezziConvenzionali.

We estimate the parameters used to define the inverse demand function in order to construct the 2020 scenarios. In particular, these 2020 parameters are computed by increasing the 2009 reference demand and prices by 22,4% and 14% respectively[9]. The emission market is limited to the Italian electricity market. Given this restriction, the total emission cap *E* computed for the 2009 scenarios corresponds exactly to the sum of the NAPs of the electricity generating companies included in the simulation tests. It amounts to about 94 Millions tons of CO_2[10]. To be in line with Directive 2009/29/EC, the 2020 cap is computed by introducing a 15% cut of the 2009 cap[11]. Finally, our mixed complementarity problems are implemented in GAMS language using PATH as solver.

4.2 *Results*

In this section, we report the results of our analysis. In particular, we want to show the effects of the EU-ETS on electricity prices, investments and generators' profits in the four scenarios presented above.

4.2.1 Impact on prices and demand

Figures 2 and 3 respectively show the trend of the 2009 and 2020 prices and demand in the four scenarios.

As expected, prices in 2020 are higher than in 2009, with the exception of the ''2020-ETS inc'' prices in some hours at the beginning of the day (see Figure 2). This happens because in this scenario, electricity is mainly served by baseload and renewables plants that contribute to lessen electricity prices. One can further notice that, both in 2009 and 2020, electricity prices are generally lower when incentives are given to investments in renewables. This is a direct consequence of the EU-ETS impact on electricity prices. As certified in the first two phases, generators are able to transfer their carbon costs in the final power prices. In the ''2009-ETS inc'' and ''2020-ETS inc'' scenarios, incentives enhance investments in low-emitting plants which contribute to lessen CO_2 allowance price compared to the respective cases without subsidies. As a result, the electricity prices in ''2009-ETS inc'' and ''2020-ETS inc'' are lower. However, in 2009 the price gap between the scenarios with and without incentives is lower than in 2020. This is mainly due to the fact that in 2020, incentives really boost investments in RES, especially in wind (see Figure

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[9] See Terna, 2010 and International Council for Capital Formation, 2005.

[10] Data are taken from the European Commission website. See http://ec.europa.eu/clima/policies/ets/allocation/index_en.htm.

[11] This 15% cut corresponds to a yearly reduction of 1,74% of the 2009 cap as foreseen by the new EU-ETS Directive.

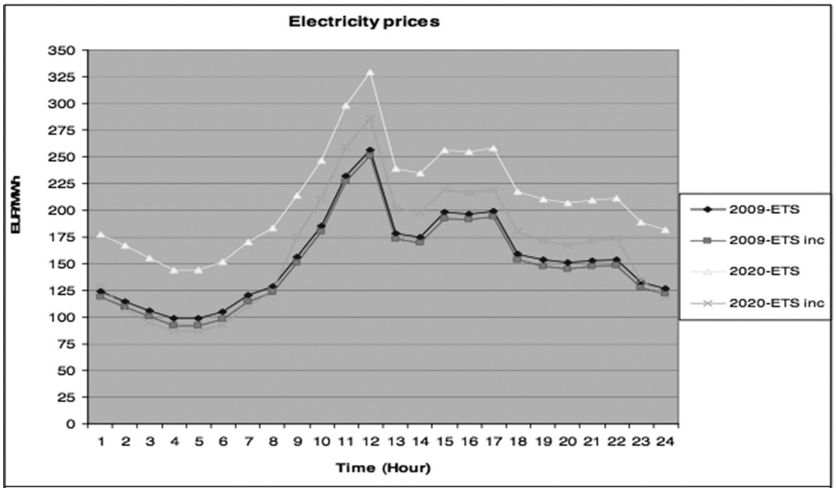

FIGURE 2. - *Average national electricity prices*

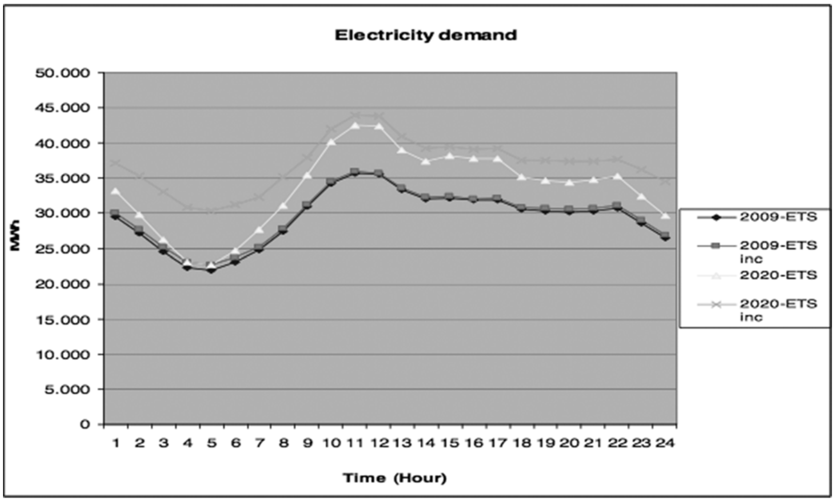

FIGURE 3. - *Average national electricity consumption*

4)[12]. In 2009, subsidies still enhance investments in wind, but in a more limited proportion[13].

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[12] In ‘‘2020-ETS inc" investments in wind totally amount to 19,705 MW compared to 8,859 MW in the ‘‘2020-ETS". In percentage, this corresponds to an increase of 122%.

[13] In ‘‘2009-ETS", generators only invest in CCGT. With incentives, they change their strategies and start building 3,791 MW of new wind plants.

Finally, demand represented in Figure 3 reflects price trends in the different scenarios. By construction, it is inversely proportional to prices.

4.2.2 Impact on investments

As already observed, the EU-ETS influences investment strategies. In Figure 4, we compare the results of the 2009 and 2020 scenarios.

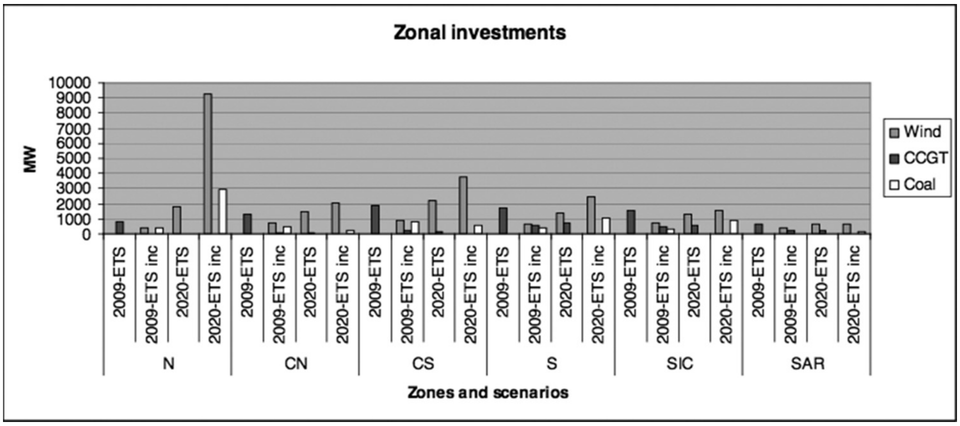

FIGURE 4. - *Zonal investments by source of energy (MW)*

In 2009, investment choices depend on incentives policies. In the ''2009-ETS'', generators only invest in CCGT. New CCGT plants are built in all zones and the total investment amounts to 7,927 MW. In ''2009-ETS inc'', the investment mix changes and includes wind (3,791 MW), CCGT (1,626 MW) and coal (2,523 MW) plants. This means that incentives make investments in renewables more accessible, but, on the other side, favor the construction of new coal plants. This happens because the reduction of the CO_2 price due to investments in RES[14] reduces the carbon costs associated to this technology that is characterized by relatively low fixed and variable costs and a high emission factor. Note that, in addition to these investments, generators also use existing capacity to produce electricity. In particular, both with and without incentives, they run all RES[15] and CCGT plants. While renewables are run at full capacity in almost all hours, existing CCGT is mainly used in the central hours of the day (from 10 a.m. to 5 p.m.).

Figure 4 also reports the results for the 2020 scenarios. The overall investment

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[14] Carbon prices amount to 39 €/MWh and 51 €/MWh in the ''2009-ETS'' and ''2009-ETS inc'' scenarios respectively. We know that these two prices are not realistic, but they can be compatible with our model assumptions. In fact, we assume that the emission market involves the electricity market only and experience has shown that the power market has been always short in emission permits.

[15] Wind, hydro, photovoltaic, and geothermic.

level in 2020 is higher than in 2009. In ‘‘2020-ETS’’, new capacity amounts to 10,638 MW of which 8,859 MW (87%) are new wind plants and the remaining 1,779 MW (17%) are CCGT. Compared to the corresponding 2009 scenario, generators reduce their global investments in CCGT in favor of wind. This happens in all zones, but it is particularly evident in the North. This change of tendency can be considered as a direct consequence of the compulsory auctioning system imposed by Directive 2009/29/EC on energy sector. In fact, during the third EU-ETS phase, power producers do not receive any free allowance for covering their CO_2 emissions. Since the aim of oligopolistic generators is to maximize their profits, they try to reduce their carbon costs by investing in renewables. Incentives given to RES enforce this tendency: investments in wind raise to 19,705 MW in the ‘‘2020-ETS inc’’. However, parallel to what observed for the ‘‘2009-ETS inc’’ scenario, this huge investment is accompanied with the construction of new coal installations (5,924 MW). The reasons of this strategy are the same as those presented for the 2009 cases.

4.2.3 Impact on generators’ profits

Table 2 analyzes the 2009 and 2020 profits by component. These components are: ‘‘Generation Revenues’’, ‘‘Generation Costs’’, ‘‘Emission Revenues’’ and ‘‘Emission Costs’’. The former two terms respectively indicate the revenues and the (fixed and variable) costs of producing electricity from new and existing plants. The latest two terms respectively point out the revenues and the costs faced by generators in the emission market. The difference between ‘‘Emission Revenues’’ and ‘‘Emission Costs’’ defines the emission opportunity costs that affect generators’ profits. More precisely, ‘‘Emission Revenues’’ corresponds to the economic value of the grandfathered allowances received by generators in the first two ETS phases, while ‘‘Emission Costs’’ is the cost of purchasing permits on the emission market. Considering the new rules imposed by Directive 2009/29/EC, in the 2020 scenarios, the column ‘‘Emission Revenues’’ is empty because during the third EU-ETS phase the power sector has to buy all needed allowances through an auctioning system[16].

Considering the figures reported in Table 2, a first remark is that profits in 2020 are higher than in 2009. This results from a combined effect of prices and demand. As shown in Figures 2 and 3, generators in 2020 produce more at a higher price. On one side, this implies an increase of the variable and fixed costs, but on the other this leads to an increase of the revenues. Comparing to 2009, power producers no longer receive permits for free, but they are able to set an electricity price that more than compensates the carbon costs, costs that are then transferred to final consumers.

The second remark is that in the ‘‘2009-ETS inc’’ scenario, generators profits

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[16] In our analysis, we do not consider the windfall profits problem, raised during the first two ETS phases, that has been extensively discussed and proved in literature (see Chen *et al.*, 2008; Kara *et al.*, 2008; Linares *et al.*, 2008; Lise *et al.*, 2010; Sijm *et al.*, 2006).

TABLE 2. - *Generators' profits in 2009 and 2020 scenarios*

Scenarios	Generation Revenues (M€)	Generation Costs (M€)	Emission Revenues (M€)	Emission Costs (M€)	Total Profits (M€)
2009-ETS	40,010	10,615	4,317	4,797	28,915
2009-ETS inc	38,997	10,779	3,268	3,631	27,854
2020-ETS	62,817	21,698	-	8,086	33,034
2020-ETS inc	53,665	17,920	-	2,578	33,166

are lower than in the corresponding case without incentives. In ''2009-ETS inc'', the raised wind penetration causes a reduction of the electricity prices. Moreover, wind technologies are characterized by high investment costs that are partially compensated by the cut in CO_2 costs. This does not happen in the ''2020-ETS inc'' scenario, because wind penetration is so high that it implies a significant reduction both in the variable generation and emission costs.

5. CONCLUSIONS

The contribution of this paper is twofold: first, we propose a theoretical description of the complementarity models and second, we illustrate and explain their mathematical structure through the use of an example. In particular, we develop a capacity expansion problem applied to the restructured Italian electricity market that is currently subject to the EU-ETS. This problem is used to measure the effects of the EU-ETS Directives on electricity prices and demand, investments and generators' profits. For this reason, we consider some investment scenarios under the CO_2 regulation with and without incentives given to RES. The scenarios also include simulations on future effects of the third EU-ETS phase on the power system.

Our analysis shows that both in the short (2009) and in long (2020) term, incentives given to RES have a double effect: on one side, they contribute to the development of wind plants, but, on the other, they lead to investments in stable but dirty technologies, like coal power plants. Without incentive policies, generators behave in a different way in the short and in the long run. In 2009, they invest only in CCGT that is a secure and efficient technology; in 2020, power producers combine investments in wind with CCGT. This technology is used to hedge the risks associated to RES. The absence of nuclear plants in the Italian power market may also influence these choices. Finally, these investment policies affect electricity prices that are generally low when RES penetration is significant.

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REFERENCES

Anderson E.J., Hu X. (2008). Finding supply function equlibria with aymmetric firms. *Operation Research*, **56**, 697-711.

Chen Y., Hobbs B.H. (2005). An Oligopolistic Power MarketModel With Tradable NO_x Permits. *IEEE*, **20**(1), 119-129.

Chen Y., Hobbs B.H., Leyffer S., Munson T.S. (2006). Leader-Follower Equilibria for Electric Power and NO_x Allowances Markets. *Computational Management Sciences*, **3**(4), 307-330.

Chen Y., Sijm J., Hobbs B.H., Lise W. (2008). Implications of CO_2 emissions trading fro short-run electricity market outcomesin northwest Europe. *Journal of Regulatory Economics*, **34**, 23-44.

Facchinei F., Pang J.-S. (2003). *Finite-Dimensional Variational Inequalities and Complementarity Problems*, Vols 1 and 2. Springer, New York.

Ferris M.C., Pang J.-S. (1997). Engineering and economic applications of complementarity problems. *Society for Industrial and Applied Mathematics*, **39**(4), 669-713.

Floro D. (2009). Selecting static oligopolistic models in the Italian wholesale electricity market. In *IEEEXplore, Proceedings of the 6th International Conference on the European Energy Market (EEM 2009)*, DOI:10.1109/EEM.2009.5207162.

Gabriel S.A., Conejo A.J., Fuller J.D., Hobbs B.F., Ruiz C. (2013). *Complementarity Modeling in Energy Markets*. Springer, New York.

García-Bertrand R., Conejo A.J. (2006). Electricity Market Equilibrium Model with Constraints Involving Prices. *International Conference on Mathematical and Statistical Modeling in Honor of Enrique Castillo, June 28-30, 2006*.

Harker P.T., Pang J.S. (1990). Finite-Dimensional Variational Inequality and Nonlinear Complementarity Problems: A survey of Theory, Algorithms and Applications. *Mathematical Programming*, **115**, 153-188.

Hartmann P., Stampacchia G. (1966). On some Nonlinear Elliptical Differential Functional Equations. *Acta Mathematica*, **48**, 161-220.

Hobbs B.H. (2001). Linear Complementarity Models of Nash–CournotCompetition in Bilateral and POOLCO Power Markets. *IEEE Transaction on Power Systems*, **16**(2), 194-202.

Hobbs B.F., Helman U. (2004). Complementarity-BasedEquilibrium Modeling for Electric Power Markets. In D. Bunn (Ed.), *Modeling Prices in Competitive Electricity Markets*, (pp. 69-98), J. Wiley.

International Council for Capital Formation, 2005. Kyoto Protocol and Beyond: the Economic Cost to Italy.

Kara M., Syri S., Lehtila A.H., Helunen S., Kekkonen V., Ruska M., Forsstrom J. (2008). The impacts of the EU CO_2 emissions trading on electricity markets and electricity consumers in Finland. *Energy Economics*, **30**, 193-211.

Linares P., Santos F.J., Ventosa M., Lapiedra L. (2008). Incorporating oligopoly, CO_2 emissions trading and greencertificates into a power generation expansion model. *Automatica*, **44**, 1608-1620.

Lise W., Sijm J., Hobbs B.H. (2010). The impact of the EU-ETS on Prices, Profits and Emissions in the Power Sector: Simulation Results with the COMPETES EU20 Model. *Environmental and Resource Economics*, **47**, 23-44.

Murphy F.H., Smeers Y. (2005). Generation capacity expansion in imperfectly competitive restructured electricity markets. *Operations Research*, **53**(4), 646-661.

Nagurney A. (1999). Network Economics. *A variational inequality approach, revised second edition. Advances in computational economics*, Vol. 10. Kluwer Academic Publishers, Boston.

Neuhoff K., Keats K., Sato M. (2006a). Allocation, incentives and distortions: the impact of EU-ETS emission allowance allocations to the electricity sector. *Climate Policy*, **6**(1), 73-91.

Neuhoff K., Ahman M., Betz R., Cludius J., Ferrario F., Holmgren K., Pal G., Grubb M., Matthes F., Rogge K., Sato M., Schleich J., Sijm J., Tuerk A., Kettner C., Walker N. (2006b). Implications of announced phase 2 national allocation plans for the EU-ETS. *Climate Policy*, **6**(5), 41-422.

Oggioni G., Smeers Y. (2009). Evaluating the impact of average cost based contracts on the industrial sector in the European emission trading scheme. *CEJOR*, **17**(2), 181-217.

Ortega J.M., Rheinboldt W.C. (1970). *Iterative Solution of Nonlinear Equations in Several Variables*. Academic Press, New York.

Pineau P.O., Murto P. (2003). An oligopolistic investment model of the Finnish electricity markeS. *Annals of Operations Research*, **121**, 123-148.

Reinaud J. (2003). Emission trading and its possible impacts on investment decisions in the power sector. *IEA Information Paper*.

Reinaud J. (2005). Industrial competitiveness under the European Union Emission Trading Scheme. *IEA Information Paper*.

Schweppe S.C., Caramanis M.C., Tabors R.D., Bohn R.E., (1988). Spot Pricing of Electricity. *Kluwer Academic Publishers*, Boston.

Sijm J.P.M., Neuhoff K., Chen Y. (2006). Cost pass thought and windfall profits in the power sector. *Climate Policy*, **5**(1), 61-78.

Terna (2010). Previsioni della domanda elettrica in Italia e del fabbisogno di potenza necessario. Available at http://www.terna.it/default/Home/SISTEMA_ELETTRICO/statistiche/previsioni_domanda_elettrica.aspx.

Terna (2011). Valori dei limiti di transito tra le zone di mercato REV.14. Available at http://www.terna.it/default/Home/SISTEMA_ELETTRICO/mercato_elettrico/Procedura_valutazione_-limiti_e_limiti_transito.aspx.

Tirole J. (1988). *The Theory of Industrial Organization*. MIT Press, Cambridge, MA.

Ventosa M., Denis R., Redondo C. (2002). Expansion planning in electricity markets. Two different approaches. *Proceedings of the 14th Power Systems Computation Conference (PSCC)*, Seville.

Ventosa M., Baillo A., Ramos A., Rivier M. (2005). Electricity market modelling trends. *Energy Policy*, **33**, 897-213.

Vives X. (1999). *Oligopoly Pricing: Old Ideas and New Tools*. MIT Press, Cambridge, MA.

Willems B., Rumiantseva I., Weigt H. (2009). Cournot versus supply functions: what does the data tell us? *Energy Economics*, **31**, 38-47.