Statistica & Applicazioni - Special Issue, 2013, pp. 57-77

DETERMINISTIC AND STOCHASTIC MODELS FOR HEDGING ELECTRICITY PORTFOLIO OF A HYDROPOWER PRODUCER

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SUMMARY

A deterministic and a stochastic multi-stage portfolio model for a hydropower producer operating in a competitive electricity market is proposed. The producer has to cope with a production function constraint imposing the scheduling of his future production. The portfolio includes its own production, a set of energy contracts for delivery or purchase, including derivatives contracts as forwards with physical delivery to hedge against risks. The goal of our models is to maximise the profit of the producer and reduce the economic risks connected to the fact that energy spot and forward prices are highly volatile. We alternatively derive the forward price by the spot dynamics and model forward dynamics obtaining consistent scenarios. Our results show that forward contracts can be used for hedging purposes. Beyond profit, the convenience of using forward contracts is a more efficient use of the hydroplant, taking advantage of the possibility of pumping water and ending up with a higher final value of the reservoir. Finally, we provide performance measures of our three-stage model with respect to deterministic one.

Keywords: *Regime Switching Model, Hydroplants, Electricity forward Contracts, Stochastic Programming.*

1. INTRODUCTION

As a result of the liberalization of energy markets, generation companies are exposed to higher uncertainties. Risk management becomes a more pressing issue for electricity retailers and producers and contracts for future delivery of electricity (i.e. forwards contracts) become a tool for hedging risk. Indeed, the retailers are highly constrained by regulated tariff consumers and they are subject to risk of high electricity price.

For a hydropower producer, on a yearly horizon, it is very important to identify the optimal scheduling of its production in order to satisfy a production profile that may be particularly difficult to match in some months of the year. The production

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profile is the forecast of customers' energy demand. Therefore, for an hydropower producer is very important to use derivative contracts in order to optimize the use of hydro resources.

The hydropower producer's portfolio includes his own production, a set of power contracts for delivery or purchase including derivative contracts, like options, futures or forwards with physical delivery, to hedge against various types of risks. In this paper we develop deterministic and stochastic portfolio models for a hydropower producer operating in a competitive electricity market. The novelty of our approach consists in a more detailed modeling of the electricity derivatives contracts, leading to an effective daily hedging.

The goal of using such a model is to reduce the economic risks connected to the fact that energy spot price may be highly volatile due to various different, unpredictable reasons (i.e. very cold winter) and to the possibility of a period of scarcity of raining or snowmelting. See Dentcheva and Römisch (1998), Gröwe-Kuska *et al.* (2008), Wallaceet *et al.* (2003), Fleten and Kristoffersen (2008), Conejo *et al.* (2008) and Nolde *et al.* (2008) for discussion on the opportunity of using stochastic programming for such problems and Yamin (2004) for a review of methods for power generation scheduling. The basis risk factors include the wholesale spot and forward prices of electric energy, which are supposed to be unaffected by the decision of the utility manager, and the uncertain inflow of hydro reservoirs, see also Fleten and Wallace (2009), Vitoriano *et al.* (2000), Latorre *et al.* (2007). The model we propose differs from the one discussed in Fleten and Wallace (1998) and Fleten and Wallace (2009) as we concentrate on the advantage of using derivatives contracts and use both electricity spot prices and forward prices as sources of uncertainties, considering inflows as deterministic. We leave the inclusion of stochastic inflows to future research.

2. THE HYDRO SYSTEM

The hydro system consists of a number of cascades, i.e. sets of hydraulically interconnected hydro plants, pumped-storage hydro plants and reservoirs. It is mathematically represented by a directed graph, where nodes represent water storages (reservoirs) and arcs represent water flows (either power generation, or pumping, or spillage).

We first introduce the model of the hydroelectric system with hourly periods. Let t be the hour index, $1 \leq t \leq T$. Let J denote the set of nodes and I denote the set of arcs. The arc-node incidence matrix, whose (i,j)-entry is denoted by $A_{i,j}$, represents the interconnections among water storages and water flows in the hydroelectric system ($A_{i,j} = -1$, if arc i leaves node j; $A_{i,j} = 1$, if arc i enters node j; $A_{i,j} = 0$, otherwise; we assume no water losses). For every arc $i \in I$ and for every node $j \in J$ the following data are relevant:

- $k_i[MWh/10^3m^3]$: energy coefficient ($k_i > 0$, if arc i represents generation; $k_i < 0$, if arc i represents pumping; $k_i = 0$, if arc i represents spillage)
- $\overline{q}_i$ $[10^3m^3/h]$: maximum water flow in arc i

- $\overline{v}_j$ $[10^3 m^3]$: maximum storage volume in reservoir j
- $v_{j,0}$ $[10^3 m^3]$: initial storage volume in reservoir j
- $\underline{v}_{j,T}$ $[10^3 m^3]$: minimum storage volume required in reservoir j at the end of hour T
- $f_{j,t}$ $[10^3 m^3/h]$: \ natural inflow in reservoir j in hour t

The power producer must schedule the production of each hydro plant, which is expressed as the product of the hydro plant energy coefficient times the turbined volume in hour t, as well as the hourly pumped and spilled volumes. The decision variables of the hydro scheduling problem are

- $q_{i,t}$ $[10^3 m^3/h]$: water flow on arc i in hour t (turbined volume, if arc i represents generation; pumped volume, if arc i represents pumping; spilled volume, if arc i represents spillage);
- $v_{j,t}$ $[10^3 m^3]$: storage volume in reservoir j at the end of hour t.

The values assigned to the decision variables must satisfy the following constraints that describe the hydroelectric system:

- flow on arc i in hour t is nonnegative and bounded above by the maximum volume that can be either turbined, or pumped, or spilled
$$0 \leq q_{i,t} \leq \overline{q}_i \qquad i \in I, \quad 1 \leq t \leq T \tag{1}$$
- the storage volume in reservoir j at the end of hour t is nonnegative and bounded above by the maximum storage volume
$$0 \leq v_{j,t} \leq \overline{v}_j \qquad j \in J, \quad 1 \leq t \leq T \tag{2}$$
- at the end of hour T, the last hour of the planning period, the storage volume in reservoir j is bounded below by the minimum storage volume required at the end of the current planning period, so as to provide the required initial storage volume at the beginning of the following planning period
$$\underline{v}_{j,T} \leq v_{j,T} \qquad j \in J \tag{3}$$
- the storage volume in reservoir j at the end of hour t must be equal to the reservoir storage volume at the end of hour $t-1$ plus the sum of inflows in hour t minus the sum of outflows in hour t
$$v_{j,t} = v_{j,t-1} + f_{j,t} + \sum_{i \in I} A_{i,j} \cdot q_{i,t} \quad j \in J, \quad 1 \leq t \leq T \tag{4}$$
where $v_{j,0}$ is a data representing the initial storage volume in reservoir j. Reservoir inflows are natural inflows, turbine discharge from upstream hydro plants, pumped volumes from downstream hydro plants, spilled volumes from upstream reservoirs. Reservoir outflows are turbine discharge to downstream hydro plants, pumped volumes to upstream hydro plants and spilled volumes to downstream reservoirs. In this paper, the values of natural inflows $f_{j,t}, j \in J, 1 \leq t \leq T$, are assumed to be known with certainty.
- the value of the reservoir at the end of the horizon $V(v_{j,T})$, is a function of reservoir level that has to be specified to avoid end effects.

Start-up and shut-down costs are considered not significant.

3. ELECTRIC POWER FORWARD CONTRACTS

An electricity forward contract is an obligation to buy or sell, at a predetermined delivery price (the forward price), a specified amount of power (1 *MW* per hour) during a delivery period fixed at the issue time of the contract. In base-load contracts the delivery is 24 hours per day, while in peak-load contracts the delivery is from 8 am to 8 pm of working days. In this paper we will not consider peak-load contracts.

The forwards contracts are standardized by the following characteristics: volume, delivery period and settlement.

The energy volume is the number of *MWh* underlying the contract: for contracts with a fixed power unit of 1 *MW*, it is equivalent to the number of hours in the delivery period. As an example, for an April contract with monthly delivery period this means a total of 1 $MW \times$ 30 $days \times 24\ h/day = 720\ MWh$. The quoted forward price is the price at which the owner of the contract will buy/sell energy during the delivery period per 1MWh. The value of the contract is the product of the quotation and the volume. For each buying or selling of the contract, we consider a transaction fee, *tc* of 0.01 € per *MW* and an estimated bid-ask spread, *ba*, of 3% of the forward price. In our example the transaction fee is 0.01 € $\times$ 30 $days \times$ 24 $h/day = 7.2$ €.

The delivery periods are fixed to each of the 12 calendar months (M1, M2, .., M12), to the four quarters (Q1, Q2, Q3, Q4) of the calendar year or to the whole calendar year. There are also shorter delivery periods of one day, one week, and the week-end. For each contract we can distinguish between a trading period and a delivery period. The trading in a given contract stops when it enters the delivery period.

Another relevant characteristic is the settlement. We can distinguish between financial contracts and physical contracts. The former requires a cash settlement of forward price against the realized spot prices during the delivery period. The latter requires energy's delivery at the delivery price during the delivery period.

For modeling reasons, let us consider the accounting of a forward contract. If we buy the same contract in two consecutive days, from a modeling point of view we should introduce two variables for the open net positions since they have two different delivery prices. An efficient way to maintain a reduced number of variables is to imagine to close the contract position at the end of day and re-open it next day at new forward price, using a single variable for the open net position. The difference in the forward price is then cumulated: in practice this is equivalent to mark-to-market mechanism.

This specification is relevant to understand the optimisation model we propose in the following section, where we want to select which contract to hold and which contract to close before the delivery period. With reference to a forward contract l, whose market price is indicated by $F_{l,t}$, assume we enter in a long position at time T_l^b, $T_l^s \leq T_l^b \leq T_l^e$, where T_l^s denotes the first trading day. Assume we maintain contract l till T_l^e, the beginning of the delivery period. The delivery period ends in T_l^d and the number of days in the delivery period is $DP_l = T_l^d - T_l^e$. See Figure 1 for an illustration of all these quantities. We can decomposed the loss/gain on the contracts into two components: the mark-to-market, during the trading period, and the

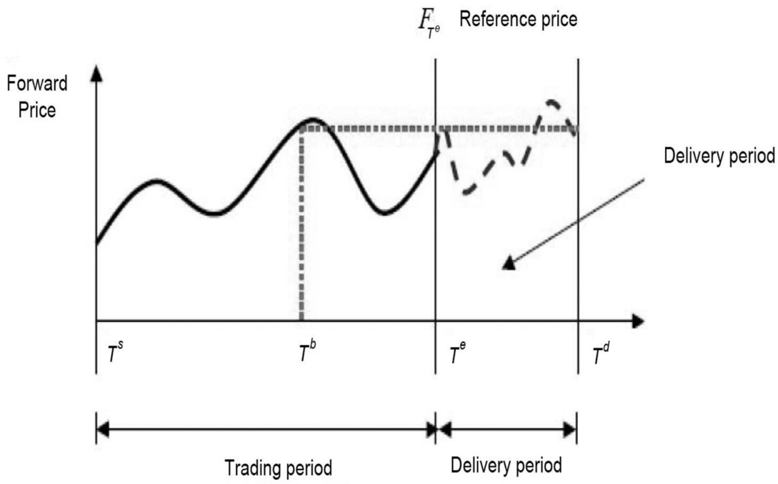

FIGURE 1. - *The Evolution of Forward Price*

settlement, during the delivery period. The first mechanism implies that from the purchase day till the last trading day, we close the position every day and immediately reopen it at the new forward price. The daily gain/loss is given by the price variations. At the end of the trading period the holder of the long position has a gain/loss of $DP_l \cdot (F_{T_l^e} - F_{T_l^b})$. The last position of the trading period is a forward with the quote $F_{T_l^e}$, called the reference price. During the delivery period we distinguish between financial and physical delivery. The former is computed considering the daily variations between the reference price and the spot price S_t. The latter consists in the payment of the reference price against the physical delivery of electricity. In equation (5) we show that this mechanism is equivalent to the classical settlement where the exchange is between the forward price fixed at the purchase of the contract and the spot price during the delivery period.

$$\sum_{t=T_l^e}^{T_l^d} (S_t - F_{l,T_l^b}) = DP_l \cdot \sum_{t=T_l^b+1}^{T_l^e} (F_{l,t} - F_{l,t-1}) + \sum_{t=T_l^e}^{T_l^d} (S_t - F_{l,T_l^e}) \tag{5}$$

If we consider physical settlement, the exchange is between the forward price fixed at the purchase of the contract and the physical delivery of energy during the delivery period.

$$-\sum_{t=T_l^e}^{T_l^d} F_{T_l^b} = DP_l \cdot \sum_{t=T_l^b+1}^{T_l^e} (F_t - F_{t-1}) - DP_l \cdot F_{T_l^e} \tag{6}$$

In this work we consider base-load contracts with physical settlement, since we decide to model the daily movements of the spot process. The introduction of peak-load contracts would require to model hourly prices.

4. THE HEDGING MODEL

The model presented in the previous section, with hourly time unit, is used by the producer for the daily scheduling. Starting from this section, we consider a yearly time horizon and a daily time unit since we want to focus on the benefit coming from the introduction of financial contracts and we want to keep the dimension of the problem tractable.

All the previous equations will be transformed to fit the new time period and t will represent the day. From now on, T represents the last day of the planning period.

The power producer is assumed to be a price taker, i.e. not able to influence the electricity market price, which is therefore exogenous to the model. We suppose to be able to hedge the producer's portfolio by buying and selling forward base-load contracts; the variable $buy_{l,t}$ ($sell_{l,t}$) is used to denote the number of positions at time t and l represents the type of contract to be bought (sold) on the forward market and $x_{l,t}$ represents the number of open positions in forward contract l at time t. Let L denote the set of forwards contracts.

The producer can buy or sell forward contracts with different delivery periods, i.e. forward contracts with weekly, monthly, quarterly and yearly delivery. If the producer has a planning horizon T of one year (expressed in days), he can decide to hedge his risk by buying or selling forward contracts with different delivery periods in the different days of his planning horizon. For contract l, T_l^e indicates the maturity of the contract, T_l^d the end of the delivery period, T_l^s indicates the time when the contract is firstly traded on the market.

We model the objective function representing the producer's profit: W_T denotes the cumulative wealth at time T and $V(v_{j,T})$ denotes the value of reservoir j at time T.

$$\max\left[W_T + \sum_j V(v_{j,T})\right] \tag{7}$$

subject to

$$\begin{aligned} W_t = {} & W_{t-1}(1+r) + S_t \cdot \sum_i k_i \cdot q_{i,t} + \sum_l \left(S_t - F_{l,T_l^e}\right) \cdot x_{l,t} \cdot I_{\left\{t\in[T_l^e,T_l^d]\right\}} + \\ & + \sum_l DP_l(F_{l,t} - F_{l,t-1}) \cdot x_{l,t} \cdot I_{\left\{t\in[T_l^s,T_l^e)\right\}} + \\ & - \sum_l (buy_{l,t} + sell_{l,t})(tc + ba \cdot F_{l,t}) \cdot I_{\left\{t\in[T_l^s,T_l^e)\right\}} \end{aligned} \tag{8}$$

for $1 \le t \le T$, with $W_0 = 0$, $F_{l,0} = 0$ for all l, $I_{\left\{t\in[T_l^e,T_l^d]\right\}} = 1$ for $t \in [T_l^e, T_l^d]$ and $I_{\left\{t\in[T_l^e,T_l^d]\right\}} = 0$ for $t \notin [T_l^e, T_l^d]$;

$$DP_l = T_l^d - T_l^e \quad \forall l \tag{9}$$

$$\sum_i k_i \cdot q_{i,t} + \sum_l x_{l,t} \cdot I_{\left\{t\in[T_l^e,T_l^d]\right\}} = D_t \quad \forall t \tag{10}$$

where D_t is the production schedule at time t, supposed deterministic and exogenous to the model. The number of open positions in forward contract l at time t, $x_{l,t}$, is recursively defined in terms of the number of open positions in forward contract l at time $t-1$, plus $buy_{l,t}$ and minus $sell_{l,t}$, where $buy_{l,t}$ and $sell_{l,t}$ are nonnegative and represent the number of contracts l bought and sold at time t respectively:

$$x_{l,t} = x_{l,t-1} + (buy_{l,t} - sell_{l,t}) \cdot I_{\{t \in [T_l^s, T_l^e)\}}. \tag{11}$$

The decision variables $q_{i,t}$ and $v_{j,t}$ represent respectively the flow on arc i in day t and the storage volume in reservoir j at the end of day t. We have to add constraints similar to (1)-(4), where the time unit t is the day.

5. THE HEDGING STOCHASTIC MODEL

In this paper we assume that the natural inflows are known with certainty and we concentrate our attention on the financial aspects, i.e. the uncertainty in the electricity spot prices and in the forward prices, and we introduce a stochastic version of the model discussed in the previous section.

A scenario tree (see for instance Dupačová *et al.*, 2000) represents the information on the daily energy spot price and contract forward price, where each path from the root to a leaf of the tree corresponds to one scenario. The stochastic model is written in terms of the nodes $\{1, \ldots, n, \ldots, N\}$ of the scenario tree and the tree structure is described by assigning to each node n the probability P_n, $1 \leq n \leq N$, and a pointer to its parent $pred(n)$, $2 \leq n \leq N$ (i.e. except the root of the tree). The planning horizon is divided in K stages, where each stage k, $1 \leq k \leq K$, is associated to the number of days T_k and to the set of nodes N_k, where $k(n)$ is the stage associated to node n. The model can be extended to any time length.

The variables $q_{i,t}$, $v_{j,t}$, $buy_{l,t}$, $sell_{l,t}$ in the deterministic model (see equations 7-11) correspond to variables $q_{i,t,n}$, $v_{j,t,n}$, $buy_{l,t,n}$, $sell_{l,t,n}$, with $n \in N_k$, if $t \in T_k$, $1 \leq k \leq K$. In this version of the model we consider only quarterly contracts. We indicate with r the risk-free interest rate, $W_{T,n}$ the cumulative wealth at time t and node n, $U(W)$ an increasing concave utility function of wealth describing the producer risk aversion.

The stochastic model finds values of the decision variables $q_{i,t,n}$, $v_{j,t,n}$, $x_{l,t,n}$, $buy_{l,t,n}$, $sell_{l,t,n}$, for $1 \leq k \leq K$, $n \in N_k$, $t \in T_k$, $i \in I$ and $j \in J$, so as to

$$\max \sum_{n \in N_K} P_n \cdot U(W_{T,n} + \sum_j V(v_{j,T,n})) \tag{12}$$

subject to

$$W_{t,n} = W_{t-1,\nu}(1+r) + S_{t,n} \cdot \sum_i k_i \cdot q_{i,t,n} +$$

$$+ \sum_l \left(S_{t,n} - F_{l,T_l^e,n} \right) \cdot x_{l,t,n} \cdot I_{\{t \in [T_l^e, T_l^d]\}} +$$

$$+\sum_{l} DP_l(F_{l,t,n} - F_{l,t-1,n}) \cdot x_{l,t,n} \cdot I_{\{t\in[T_l^s,T_l^e)\}} + \tag{13}$$

$$-\sum_{l}(buy_{l,t,n} + sell_{l,t,n})(tc + ba \cdot F_{l,t,n}) \cdot I_{\{t\in[T_l^s,T_l^e)\}}$$

for $2 \leq k \leq K$, $t \in T_k$, $n \in N_k$ and with $W_{0,0} = 0$ and $F_{l,0,1} = 0$ for all l;

$$DP_l = T_l^d - T_l^e \quad \forall l \tag{14}$$

$$\sum_{i} k_i q_{i,t,n} + \sum_{l} x_{l,t,n} \cdot I_{\{t\in[T_l^e,T_l^d]\}} = D_t \tag{15}$$

where D_t is the production scheduling at time t;

$$0 \leq q_{i,t,n} \leq \overline{q}_i \qquad i \in I,\ t \in T_k,\ n \in N_k,\ 1 \leq k \leq K \tag{16}$$

$$0 \leq v_{j,t,n} \leq \overline{v}_j \qquad j \in J,\ t \in T_k,\ n \in N_k,\ 1 \leq k \leq K \tag{17}$$

$$v_{j,t,n} = v_{j,t-1,\nu} + f_{j,t} + \sum_{i\in I} A_{i,j} \cdot q_{i,t,n} j \in J,\ t \in T_k,\ n \in N_k,\ 1 \leq k \leq K \tag{18}$$

$$\underline{v}_{j,T} \leq v_{j,T,n} \qquad j \in J, \quad n \in N_K \tag{19}$$

$$x_{l,t,n} = x_{l,t-1,\nu} + (buy_{l,t,n} - sell_{l,t,n}) \cdot I_{\{t\in[T_l^s,T_l^e)\}} \qquad t \in T_k, \quad n \in N_k \tag{20}$$

$$buy_{l,t,n} \geq 0 \ , \qquad sell_{l,t,n} \geq 0 \qquad l \in L,\ t \in T_k,\ n \in N_k,\ 1 \leq k \leq K \tag{21}$$

The objective function (12) is the expected utility of the sum of final wealth and the end-of-period value of the reservoirs. Equation (13) defines the cumulative wealth at time t and node n. It consists of the following terms:

1. the capitalised previous day wealth;
2. the revenue obtained by selling, at spot price, the produced energy;
3. the gain/loss from the forward contracts that have reached their delivery period;
4. the gain/loss from the mark-to-market-like operations for the contracts in their trading period;
5. the transation costs and bid-ask spread differential of the trading operations.

In the wealth equation (13), in the mass balance equations (18) of the hydro system model and in the financial balance equation (20) $\nu = n$, if $t-1, t \in T_k$, and $\nu = pred(n)$, if $t-1 \in T_{k-1}$ and $t \in T_k$. The constraints from (14) to (19) and constraint (21) are the equivalent of the deterministic constraints related to the structure of the hydroplant and to the forward contracts.

6. SPOT AND FORWARD ELECTRICITY PRICES

6.1 *Modelling the electricity spot prices*

The Italian electricity spot market was opened in 2003, its activity has been increasing during the last years and can be considered as a liquid market with many daily transactions. In our analysis we consider the daily base-load spot prices time series from 1/1/2008 to 9/9/2009. After removing the daily and weekly seasonal components, we analyze the log prices data and we find stationarity but no strong presence of spikes: only four observations are larger then 3 times the standard deviation on the whole period. The log spot price exhibits autocorrelation, heteroschedasticity but not a dramatic kurtosis. In line with recent findings in literature, we fit a regime switching model able to capture different market conditions, in terms of changing mean and volatilities.

We assume that y_t, the log price process, follows an AR(1) model depending on the state variables s_t:

$$y_t = \mu_{s_t} + \Phi_{s_t} y_{t-1} + \varepsilon_t \quad \text{where} \quad \varepsilon_t \sim i.i.d\ N(0, \sigma^2_{s_t}) \tag{22}$$

where s_t changes through time and takes values $j = 1, \dots, J$. The changes in s_t are described by a Markov chain $P(s_{t+1} = j | s_t = i) = p_{ji}$. We do not observe s_t directly, but we only infer it through the observed behavior of y_t.

Using the complete data set we find the evidence of the presence of two regimes (see Figure 2). The parameters needed to fully describe the probability law governing y_t are then the volatility of the Gaussian innovation, the autoregressive coefficients, the two intercepts and the two state transition probabilities, p_{11} and p_{22}. In Tables 1 and 2 we present the estimated parameters with the t-statistics in parenthesis.

TABLE 1. - *Table related to Figure 2*

	μ	Φ_{s_t}	σ
State 1	0.0140 (2.4140)	0.1510 (1.9610)	0.0760 (18.5366)
State 2	−0.0210 (−1.8584)	−0.0507 (0.6225)	0.1409 (19.8451)

TABLE 2. - *Transition probability between the two regimes in Figure 2*

Transition	State 1	State 2
State 1	0.96 (0.05)	0.04 (0.02)
State 2	0.05 (0.03)	0.95 (0.04)

For a multistage stochastic programming model a scenario tree is needed where information is revealed. Hence, we generate 100 independent scenarios describing the evolution of spot prices on a time horizon of one year and we aggregate them in

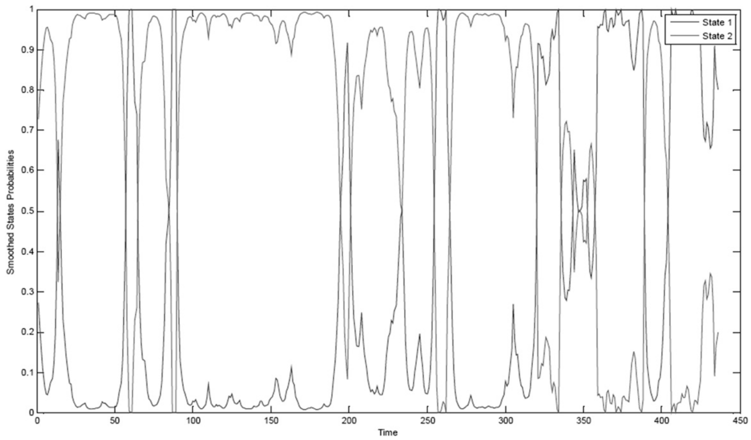

FIGURE 2. - *The Spot Switching Model*

a recombining tree using the backward scenario reduction technique proposed by Pflug (2001), and Pflug and Hochreiter (2007). By reducing the 100 scenarios we obtain a three-stage tree and P_ξ, the probability of each scenario ξ. In order to maintain consistency with the market, the spot prices scenarios are adjusted so that the expected average spot price in a period is equal to the current market price of a forward with delivery in that period.

To avoid arbitrage opportunities we impose that spot price tree satisfies the martingale property. To this purpose we introduce a correction on the drift preserving the variance in each point along time on the tree

$$S_\xi(t) = \tilde{S}_\xi(t) - \sum_\xi P_\xi \cdot \tilde{S}_\xi(t) + \tilde{S}(t_0) \quad \forall t \tag{23}$$

where $\tilde{S}_\xi(t)$ is the spot price along the scenario ξ at time t generated from the simulation procedure and $S_\xi(t)$ is the corrected spot price along the scenario ξ at time t.

6.2 *Modelling the electricty forward prices*

The theoretical dynamics of the forward prices can be derived from the spot dynamics. The price at time t along the scenario ξ of the forward contract with delivery period $[T_1, T_2]$ can be constructed on the spot scenarios tree as the conditional expected value of the average spot prices in the delivery period computed under an equivalent martingale measure on the subtree S_t generated in the node following t.

$$F_\xi(t, T_1, T_2) = \sum_{\eta \in S_t} P_{t,\eta} \cdot \frac{\sum_{\tau=T_1}^{T_2-1} S_\eta(\tau)}{T_2 - T_1} \tag{24}$$

where $S_\eta(\tau)$ is the spot price in scenario η at time τ, $P_{t,\eta}$ is the conditional probability of the scenario η at time t and $F_\xi(t, T_1, T_2)$ is the conditional forward price in t for a contract with delivery period in $[T_1, T_2]$ along scenario ξ.

In a complete market this measure is unique and it assures a unique arbitrage free price of the forward. In incomplete markets, as the electricity market, this measure is not unique. A possible approach is to remain under the real probability measure and estimate and incorporate a market price of risk in the drift. However, this approach requires a liquid market for forward contracts.

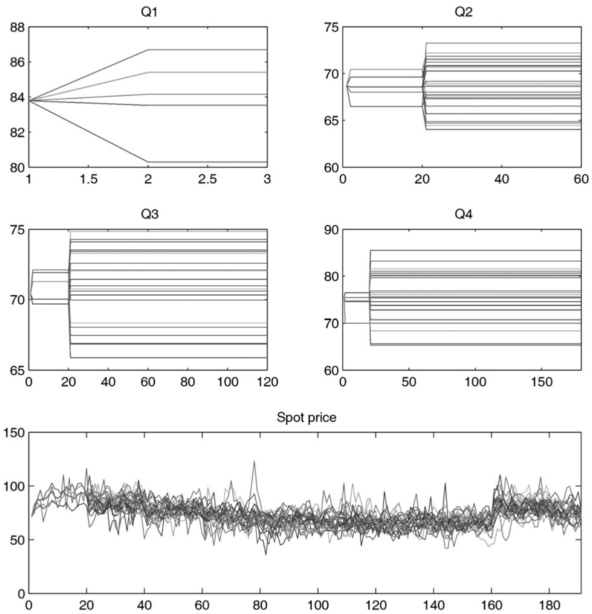

FIGURE 3. - *Scenarios for spot and related forward prices*

Alternatively, the theoretical dynamics of the forward prices can be derived modelling directly the dynamics of the forward prices. The motivation of this approach is that, in reality, the electricity spot and futures prices are not closely related, as it is typical for other commodities, such as crude oil. This happen due, among other things, to the difficulty to store electricity. Water reservoirs can be used for storing potential energy in limited quantities due to the their physical capacity. Electricity spot and forward prices can be very far from each other. Often, the spot and futures markets are so dissimilar that the relationship between spot and futures prices breaks down. If we compare, ex post, the forward quotations with the realised spot prices we observe that the difference does not necessarily tend to zero for contracts approaching their maturity.

''For example, for the Nordpool data, the historical correlation (computed using a moving window of the past 60 days) between the electricity spot and the nearby futures price ranges from 0.65 to -0.15, indicating that the futures price is a poor proxy for the electricity spot price.'' see Borovkova and Geman (2006), Borovkova and Geman (2006a).

For this reason we model explicitly the dynamics of the forward curve imposing that the scenarios generated for the spot and forward contracts are correlated and consistent with the today observed contracts quotations, i.e we impose that

1. the spot scenarios are consistent with the observed quotations of the forward contracts (24) and
2. the forward price and the spot price are martingales.

To this purpose we introduce a correction on the drift preserving the variance in each point along time on the tree

$$F_\xi(t, T_1, T_2) = \tilde{F}_\xi(t, T_1, T_2) - \sum_\xi P_\xi \cdot \tilde{F}_\xi(t, T_1, T_2) + \tilde{F}(t_0, T_1, T_2) \quad \forall t \tag{25}$$

where $\tilde{F}_\xi(t, T_1, T_2)$ is the forward price along the scenario ξ at time t generated from the simulation procedure and $F_\xi(t, T_1, T_2)$ is the corrected forward price along the scenario ξ at time t. The same procedure is used for the spot price on the tree. This second approach implies that the sources of stochasticity are two: the spot dynamics and the forward curve dynamics.

6.3 *The electricity forward curve*

The electricity forward curve is a non-trivial object and requires special attention, mainly for two reasons. Firstly, forward contracts are subject to a seasonal effect related to the delivery period. Secondly, the term structure cannot be constructed simply by interpolating between points in the price maturity space because electricity forward contracts concern delivery of electricity during a specific time interval – week, month, year – in the future. Consequently, the methods developed for fixed income markets cannot be applied directly to electricity price data.

We restrict our analysis to one segment of the term structure, the quarterly contracts, and leave a more comprehensive analysis for future research. We apply the model to the forward prices of quarterly contracts in the period 20/10/08-9/09/09 on Italian data of Over-The-Counter (OTC) contracts quoted at the TFS (Tradition Financial Services). TFS is a private platform that trades the full spectrum of OTC energy derivatives, and, at present, it is more liquid than the standardised market, the IDEX (Italian Derivatives Energy Exchange) just started at the beginning of 2009.

The idea is to compute from the daily quotations of the forward contracts, the forward term structure for fixed key rates and analyse the dynamics of the term structure.

In order to derive the forward curve we have to remove the seasonal effect associated with forward contracts. We follow Borovkova and Geman (2006), Borovkova and Geman (2006a). They observe that seasonal effects in the spot price and in the futures contracts are significantly different and that the main feature of electricity forward curves is the seasonality attached to the delivery period, not to the trading day. Let $F(t, T, T+Q)$ be the observed day-t price of the forward contract expiring in T with $T = (T1, T2, T3, T4)$. We fix the beginning of the delivery period at 4 dates (January 1st, April 1st, July 1st and September 1st) and the length Q of delivery period to 3 months in order to represent the quarterly contracts $Q1, Q2, Q3, Q4$.

We estimate the deterministic seasonal forward premium, $\pi(T)$, for each delivery date assuming that the forward price is the product of three component: the average level of the forward curve or the average forward price prevailing at date t, which we denote by $\bar{F}(t)$, a seasonal component (the premium $\pi(T)$) and $\gamma(t, T-t)$, a component which depends on the time to maturity

$$F(t, T, T+Q) = \bar{F}(t)e^{\pi(T)-\gamma(t,T-t)(T-t)}. \tag{26}$$

Note that we have separated the dependence on maturity date T (or, rather, maturity calendar quarter) from the dependence on time to maturity $\tau = T - t$. The maturity date (maturity calendar month) influences the futures price via the seasonal premium $\pi(T)$, while the effect of time to maturity τ enters the futures price via $\gamma(t, T-t)$.

$\bar{F}(t)$ is computed by

$$\ln(\bar{F}(t)) = \frac{1}{4} \sum_{T=T1,\dots,T4} \ln(F(t, T, T+Q)). \tag{27}$$

The seasonal premium $\pi(T)$ is defined as the average deviation from the mean value of the log forward quotations

$$\hat{\pi}(T) = \frac{1}{n} \sum_t (\ln(F(t, T, T+Q)) - \ln(\bar{F}(t))) \quad T = T1, \dots, T4 \tag{28}$$

where n is the length of time series of forward contracts expiring in T.

The deseasonalised price at time t, $F^{DS}(t, T-t)$, depends only on the time to maturity $T - t$ and not on the delivery period

$$F^{DS}(t, T-t) = F(t, T, T+Q)e^{-\pi(T)} \quad T = (T1, T2, T3, T4). \tag{29}$$

The estimated seasonal premia for quarterly electricity forwards are shown in Figure 4.

As expected, forwards expiring in winter are at a premium with respect to the average price level, and summer forwards at a discount: the January premium is the highest, at 15%, while the April is -12%.

Once we have removed the seasonal component we can construct the term structure forward key rates according to time of maturity. In our application we have the time series related to quarterly contracts on one year (from October 2008 to September 2009). The contract delivering in $T1$ for each year is traded in the last quarter of the previuos year. The contract delivering in $T2$ is traded in the first quarter of

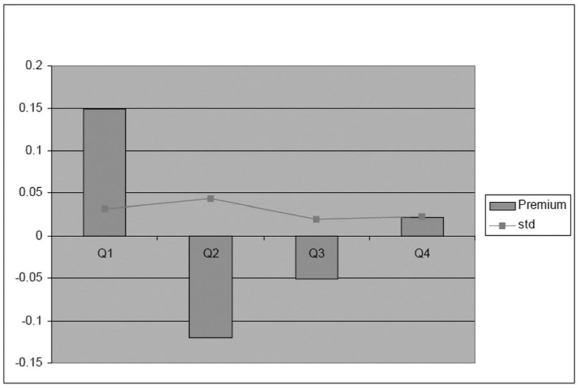

FIGURE 4. - *The forward seasonal premia estimated from 20/10/2008-9/9/2009 quarterly forward prices*

the year and so on. Therefore, the forward key rate related to the next quarter, $F^{DS}_{Q_1}(t)$ is obtained by $F^{DS}(t, T_1 - t)$ if $0 < T_1 - t < Q$ or by $F^{DS}(t, T_2 - t)$ if $0 < T_2 - t < Q$ and so on. The forward key rate related to the second quarter $F^{DS}_{Q_2}(t)$ is obtained by $F^{DS}(t, T_1 - t)$ if $Q < T_1 - t < 2Q$ or by $F^{DS}(t, T_2 - t)$ if $Q < T_2 - t < 2Q$ and so on.

We concentrate our attention on four key rates relative to the four quarterly maturities. The information on the daily deseasonalized historical prices of forward key rates $Q1, Q2, Q3, Q4$ are constructed according to the time to maturity of the traded contracts. Assuming Q_i can take the values $Q1, Q2, Q3, Q4$, the forward key rate related to Q_i

$$F^{DS}_{Q_i}(t) = \sum_{j=1}^{4} F^{DS}(t, T_j - t) 1_{\{(i-1)Q < T_j - t < iQ\}} \tag{30}$$

and the related rate of return is

$$r^{DS}_{Q_i}(t) = ln(F^{DS}_{Q_i}(t)/F^{DS}_{Q_i}(t-1)). \tag{31}$$

6.4 *Spot and forward prices scenario generation*

The procedure used in the forecasting approach for scenario generation involves two distinct steps. By Principal Component Analysis (PCA) on the daily deseasonalized historical returns of forward key rates, we compute the orthogonal factors. The first three factors explain 92% of the forward curve and correspond to a parallel shift (first factor which explains 61%), a tilting (second factor which explains 17%) and a curvature effect (third factor explains 14%), see Figure 5. We consider as relevant the first two factors that have an explanatory power of about 80% of the variability.

The residuals $\varepsilon_{i,t}$ contain all the information related to the remaining 20% of variability and the correlation among the returns of the different maturities.

We model

$$r_{Q_i}^{DS}(t) = \sum_{j=1}^{2} a_{i,j} f_{j,t} + \varepsilon_{i,t} \tag{32}$$

where $a_{i,j}$ is the factor loading of factor j for key rate i and $f_{j,t}$ if the value of factor j at time t and $r_{Q_i}^{DS}(t)$ are computed through equativo (31). Thus, from (32), we compute the residuals $\varepsilon_{i,t}$ not explained by the first two factors obtained from the PCA and we then model the variance of the residuals with a GARCH model in order to capture the dependence of returns. This model incorporates:

- a dependence effect given by the relevance of the observations of the immediate past (conditional term)
- a feedback mechanism through which past observations are taken into consideration to explain the present volatility value (autoregressive part).

Roughly speaking, if a time series exhibits GARCH effects, it means it is heteroskedastic; that is, its variance may be well described by a time-varying process.

The variables to be modeled are the residuals series obtained by the PCA. We applied the following univariate GARCH(1,1) to each of them:

$$\varepsilon_{i,t} = z_t h_{i,t} \tag{33}$$

$$h_{i,t}^2 = \alpha_1 \varepsilon_{i,t-1}^2 + \alpha_2 h_{i,t-1}^2 \tag{34}$$

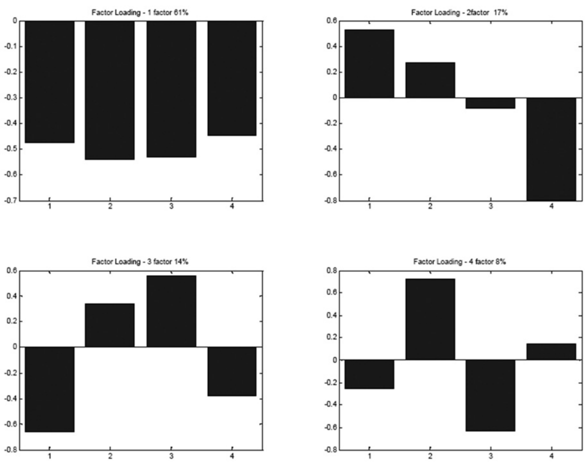

FIGURE 5. - *Factor loadings $a_{i,j}$ obtained by PCA on the deseasonalized quarterly forward prices*

where $h_{i,t}^2$ is the conditional variance process of the residuals of Q_i and $\varepsilon_{i,t}$ is the innovation of the time series process, with $\varepsilon_{i,t} = z_t h_{i,t}$ and z_t is a Gaussian *i.i.d*. process with zero mean and unit variance. In Table 3 we report the estimate of the GARCH(1,1) models and the corresponding asymptotic *t*-statistics.

TABLE 3. - *Garch(1,1) parameters and t-statistics*

	α_1	α_2
ε_1	0.0376 (3.2478)	0.9499(54.0736)
ε_2	0.0233 (3.3218)	0.9654(97.9361)
ε_3	0.1274 (2.7203)	0.8492(21.4536)
ε_4	0.1125 (1.6657)	0.8589(13.5745)

To summarize, the principal components, constant in time, reflect the dependence of the forward prices for different maturities, while the GARCH model is devoted to the dependence of the forward prices in time.

Finally, in order to generate correlated scenarios, we combine together the standardized residuals of the GARCH(1,1) model and the residuals from the regime switching model for the same days. We do not impose any parametric assumption on the marginal distributions and use the the empirical cumulative distribution to fit a Gaussian copula of the historical residuals vectors. We simulate a vector of corre-

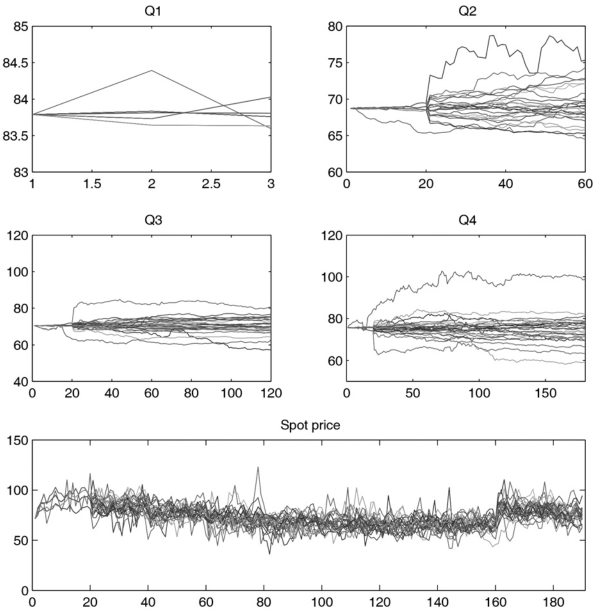

FIGURE 6. - *Multivariate scenarios for spot and forward prices*

lated innovations from the Gaussian copula and reconstruct the forecasted scenarios using the estimated principal factors for the forward return scenario and the regime switching for the log spot price, finally adding the seasonal premia. By this procedure we generate correlated scenarios for spot and forward prices. Hence, we generate 100 correlated scenarios and we aggregated them in a recombining tree using the backward scenario reduction technique proposed by Pflug and Hochreiter (2007) (see Figure 6). Finally, we adjust the multivariate tree, as described above, in order to guarantee market consistency.

7. NUMERICAL RESULTS

In this section we discuss the numerical results obtained by solving the stochastic model on two cases studies. The simulation framework is based on MATLAB release 12 and on GAMS release 21.5, for modeling and solving the optimization problem by non linear optimization package (MINOS).

The hydro system is composed by one cascade with three basins and three hydro plants, one of these is a pumped storage hydro plant as shown in Figure 7 (see Tables 4 and 5 for input data of the hydro system). In order to represent the scenarios we introduce the following notation, see also Vespucci *et al.* (2012), $T_1 = 1$, $T_2 = \{t : 2 \leq t \leq 20\}$, $T_3 = \{t : 21 \leq t \leq 191\}$. We have considered twenty five scenarios represented by means of a scenario tree where the nodes are as follows: $N_1 = 1$, $N_2 = \{2, \ldots, 6\}$, $N_3 = \{7, \ldots, 31\}$.

TABLE 4. - *Hydro basin data: capacity, initial and minimum final storage volumes*

basin	$\overline{v}_j$ (10^3m^3)	$v_{j,0}$ (10^3m^3)	$\underline{v}_{j,T}$ (10^3m^3)
v_1	1000	100	0
v_2	2000	1000	500
v_3	2000	1000	500

TABLE 5. - *Hydro arc data: energy coefficient and capacity*

arc	k_i	$\overline{q}_i$ (10^3m^3)/h
c_1	1.0	100
c_2	−1.7	50
c_3	1.3	150
c_4	0.9	120

In order to assess the value of modeling uncertainty for our three-stage problems, we follow the procedure introduced in the literature by Escudero *et al.* (2000) and Vespucci *et al.* (2012) for evaluating the value of the stochastic solution for three-stage problem, and further refined in Maggioni *et al.* (2012).

The procedure is based on the idea of reproducing the decision process as the uncertainty reveals: this procedure is suitable for multistage problems.

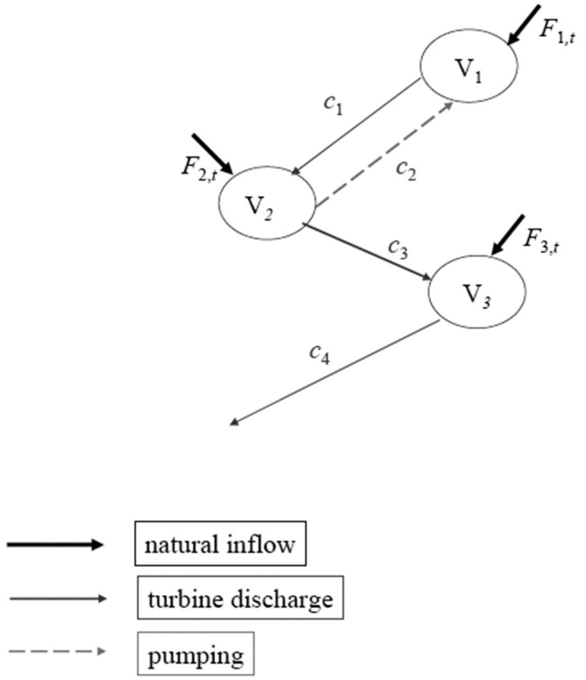

FIGURE 7. - *The hydro system*

The optimal objective value obtained in stage 3 is called *Multistage Expected Value of the Refrence Scenario* (MEVRS), where the reference scenario is the mean value scenario. Technically, this is computed as follows.

1. Scenario tree $T_{1,mean}$, is defined by considering the expected value of the uncertanty parameters (spot and forward prices); the stochastic model with scenario tree $T_{1,mean}$ is solved and the optimal values of the first stage variables are stored. In this way the optimal solution of the *Expected mean Value* (EV) problem is computed.
2. Scenario tree $T_{2,mean}$ is defined by evaluating the expected value of the spot and forward prices on nodes belonging to N_3. The stochastic model with scenario tree $T_{2,mean}$ is solved having assigned the value stored at step 1. to the first stage decision variables. The optimal value of second stage variables are stored.
3. The stochastic model on benchmark tree T_1 is solved, assigning to the first stage decision variables the values stored at step 1 and to the second stage decision variables the values stored at step 2.

Firstly, we have solved the model considering only one source of variability, the spot price. We notice a decrease both in the expected value and in the variance of the cumulated profit along the scenarios. It is natural since the contracts are used for hedging the risk related to movements of the spot prices. The main difference between the solutions with and without forward contracts is in the use of production scheduling. The introduction of forward contracts leads to a more efficient use of pumping, ending up with a higher expected value of water at the time horizon T, see Giacometti *et al.* (2011) for more details: it means that the forward contracts allows you more flexibility in managing the producer's production schedule. Overall, the effect of using forwards is an increase of the value of the objective function – from 267,452.00 to 285,248.42 €.

Finally, we consider all the sources of variability, the spot and the forward prices. If we consider only the financial part, we notice an increase both in the expected value of the profit and in the variance of the cumulated wealth along the scenarios. In this case we do not have pure hedging but speculative contracts. As before, the use of forward contracts allows a more efficient use of water pumping with a higher final level of water in the basins. Overall, the effects of the financial part and the production scheduling lead to an increase in the objective function, from 232,068.61 to 251,308.03 €. We report the certainty equivalent $U^{-1}(E[U(\cdot)])$ obtained by using a power utility function $U(\cdot)$ with risk aversion coefficient -0.5, where $E[\cdot]$ is the expected value operator.

We observe in Table 6 that the value of the objective function (EV value) using one single average scenario of respectively spot and forward prices (the deterministic case) is higher than in the stochastic solution. We also compute the Multistage expected value of the mean value scenario (MEVRS = 244,509.78). The modified value of the stochastic solution (Modified VSS = MEVRS-RP) is 6,798.25 and it allows us to obtain the goodness of the expected solution value when the expected values are replaced by the random values for the input variables. High values of Modified VSS indicate the advantage of using the stochastic approach with respect to the deterministic one.

TABLE 6. - *Two sources of stochasticity*

Profit Value (Euro)	Certainty equivalent
Deterministic Model (EV)	21,248.62 €
Stochastic Model (RP)	251,308.03 €
Multistage expected value of the mean value scenario (MEVRS)	244,509.78 €
Modified VSS (MVSS)	6,798.25 €

8. CONCLUSIONS

In this paper we have introduced a model for the daily hydro-power system scheduling problem with scenarios on daily power production. The model is a stochastic multi-stage non linear model where the profit comes from the direct production and from buying/selling base-load contracts with physical settlement in the forward energy market. We consider as random variables the spot energy price and the forward prices.

When we consider only one source of stochasticity the forward contracts can be used for hedging purposes. When we consider two sources of uncertainties, we allow for a speculative behavior of the producer. Our results show that, apart from the financial gains, the convenience of using derivatives contracts is a more efficient use of the hydroplant, taking advantage of the possibility of pumping water and ending up with a higher final value of the reservoir.

ACKNOWLEDGEMENTS

The authors greatly acknowledge the support of Bergamo University research grants 2009-2010-2011 (coordinator: M. Bertocchi), research grants 2010-2011 (coordinator: R. Giacometti) and research grant Regione Lombardia ''Metodi di integrazione delle fonti energetiche rinnovabili e monitoraggio satellitare dell'impatto ambientale'', EN-17,ID 17369.10, 2010.

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