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STRUCTURAL TIME SERIES MODELS FOR LEVEL AND VOLATILITY OF HOURLY ELECTRICITY PRICES

Paolo Chirico*

SUMMARY

This study considers an empirical investigation of the hourly PUN, which is the spot price of a megawatt on the Italian electricity market. This price is characterised by strong intra-day seasonality, i.e., hourly effects, which influence the level and volatility price, and many parameters are involved in the ARMA-GARCH modelling process. In order to reduce the number of parameters, an alternative modelling approach is presented based on structural time series modelling, i.e., an autoregressive model with seasonal effects at the PUN level and a stochastic volatility model (with seasonal effects) for the PUN volatility. This modelling approach allows us to treat seasonality as a latent stochastic component, which is governed by only a few parameters. The results obtained using this method demonstrate that the proposed modelling approach has beneficial strengths, and thus it should be developed further.

Keywords: *Hourly Electricity Price, Stochastic Volatility Model, Structural Time Series Model.*

1. INTRODUCTION

Similar to most OECD countries, electricity is traded in an electricity day-ahead market in Italy. In particular, the Italian electricity market is a *pool* market because market clearing and scheduling are coordinated by an authority called the Energetic Markets Manager, GME, (Cervigni and Perekhodtsev, 2013). Every day, market participants submit their production offers and their consumption bids for electricity during each hour on the following day to the GME. The clearing price for each hour is the price that makes the supply for that hour equal to the demand. The market clearing price ensures the maximization of the gains from the trade and the efficiency of electricity dispatch (i.e., the organization of electricity flows onto the electricity net).

Electricity markets are conceptually similar to financial and commodities markets, but the price formation is more complex because of the singular features of electricity. Indeed, electricity cannot be stored in large quantities, electricity production can be affected by weather conditions (aeolian and photovoltaic production), and it is not efficient for big electricity plants to vary their production flows rapidly. Therefore, electricity prices possess some of the typical features of financial prices (e.g., volatility clustering), but also specific features such as seasonality, jumps and peaks, and mean reversion, which differentiate electricity prices from other financial and com-

* Dipartimento di Economia e Statistica - Università di Torino - Lungo Dora Siena, 100 - 10153 TORINO (e-mail: paolo.chirico@unito.it).

modity prices (see Bunn, 2004, for a broad review of the features of electricity prices).

Seasonality refers to the presence of periodic patterns in the level and the volatility of electricity prices, which are due to the dependence of the demand for electricity on human activities and weather conditions, e.g., the demand is higher in the afternoon and lower at night, as well as being higher on working days and lower at the weekend.

Jumps and peaks are due to difficulties with electricity dispatching. Thus, shocks in the demand or in the supply of electricity may occur for several reasons such as technical problems with the electricity net (flow congestion) or unexpected peaks in demand due to changes in the weather conditions. However, these shocks do not change the price trend, which tends to remain at the average level during the short to middle term (mean reversion).

For these features, the prediction of electricity prices and their volatility has been a demanding challenge for several scholars since the origin of electricity markets (see Weron, 2014, for a recent review of the principal essays). In particular, hourly electricity prices (HEPs) have been hard to treat because spikes/jumps and seasonality are stronger at hourly frequency than other frequencies; moreover, these prices are not a univariate stochastic process, but a 24 dimensions multivariate process. Actually, the prices of hall the hours of the next day are known at the same time, not one hour at a time. Therefore, HEPs has been principally modelled by panel models (Huisman, Huurman and Mahieu, 2007; Peña, 2012) or VAR models (Raviv, Bouwman, van Dijk, 2015). Nevertheless, this way to model HEPs has to face some complications: the use of a lot of parameters, the single processes can be co-integrated, the loss of significant information about intra-day links. Moreover, we may interested in some latent variables related to HEPs like seasonality (hourly effects) or volatility whose determination/extraction could be better realized one hour at a time than simultaneously every 24 hours. Actually, the extraction of each hourly effect should take into account the hourly effects in the previous 24 hours.

Since this study focus principally on the analysis of seasonality affecting HEPs and their volatility, it was useful to consider HEPs as a univariate process even if they are not in practice. Then,a structural time series model was used to filter hourly effects from HEPS of Italian wholesale electricity market in order to analyse seasonality and non-seasonal prices separately. A similar structural model was apply to draw the periodic volatility of HEPs.

Structural time series models are very suitable to detect possible trends in the structural components of electricity prices (Chirico, 2016). For example, analysing the hourly effects series, we can note that the hourly effects change over the months and seasons, because the HEPs depends on human daily activities which change over month and seasons. That information can be taken into account to develop long-term forecasting models.

The rest of paper is organized as follows: in Section 2 structural time series models are presented for HEPs and for the volatility of HEPs; Section 3 illustrates the application of the models to the hourly prices cleared by the Italian wholesale electricity market; finally, the Section 4 reports the study Conclusions.

As stated above, the hourly price of electricity is characterised by a strong periodic pattern with a periodic length of $S = 24$, so the hourly price p_t can be viewed as the sum of two components: (i) the price in the absence of seasonality, p_t^{ns}, and (ii) the effect of seasonality (hourly effect), s_t:

$$p_t = p_t^{ns} + s_t$$

A basic model for the non-seasonal price can be defined as follows:

$$p_t^{ns} = \overline{p}_t + \varepsilon_{0t}$$
$$\overline{p}_t = \delta + \sum_{j=1}^{k} \phi_j \overline{p}_{t-j} + \varepsilon_{1t}$$

where $\overline{p}_t$ is the level of the non-seasonal price, and ε_{0t} and ε_{1t} are white-noise errors (disturbances). More specifically, ε_{0t} represents shocks which produce effects only on the current price; ε_{1t} represents shocks that cause a structural effects on the level price, and then have effects on the future prices.

Since the most electricity price often exhibits mean reversion, $\overline{p}_t$ is a mean stationary process, so $\sum_{j=1}^{k} \phi_J < 1$ (but not much lower than one) may be expected (Knittel and Roberts, 2005).

With respect to seasonality, it is known that the HEPs exhibit a main periodic pattern with length $S = 24$, but also a weekly ($S = 24 \times 7$) pattern, so each hourly effect is not exactly constant every 24 hours, and thus the hourly effects within a day are generally different from those in the following or previous days. A practical way to treat this variability is by assuming that the daily seasonality in the hourly prices is a stochastic component, i.e., the sum of 24 consecutive hourly effects is not zero as found in the case of deterministic seasonality (constant hourly effects), but instead it is a stochastic zero mean error:

$$\sum_{j=0}^{23} s_{t-j} = \varepsilon_{2t}$$

The previous equations comprise the following structural time series model:

$$\begin{aligned} p_t &= \overline{p}_t + s_t + \varepsilon_{0t} \qquad && \varepsilon_{0t} \sim n.i.d(0, \sigma_0^2) \\ \overline{p}_t &= \delta + \sum_{j=1}^{k} \phi_j \overline{p}_{t-j} + \varepsilon_{1t} \qquad && \varepsilon_{1t} \sim n.i.d(0, \sigma_1^2) \\ s_t &= -\sum_{j=1}^{23} s_{t-j} + \varepsilon_{2t} \qquad && \varepsilon_{2t} \sim n.i.d(0, \sigma_2^2) \end{aligned} \tag{1}$$

where δ, ϕ, σ_0, σ_1, σ_2 are unknown parameters.

The model above can be referred to as *autoregressive level* model with seasonal effects; it is formulated in the state space form, and, under the assumption of gaussian disturbances, the price forecasts as well as the estimates of the structural series ($\overline{p}_t$, s_t) and the model parameters (σ_0^2, σ_1^2, σ_2^2) can be obtained using the Kalman filter (Harvey, 1990). The assumption of Gaussian disturbances might be not realistic since ε_{0t} and ε_{1t} include spikes and jumps which make their distributions havy-tailed. In this case the Kalman filter will only yield Linear MMSE estimators of the future prices rather than MMSE ones. Moreover the exact likelihood cannot be obtained from the resulting prediction errors, so the forecasts are quasi-maximum likelihood (QML) estimates. However, if the disturbance distributions are not too far from normality, Kalman filter works quite well (Ruiz, 1994).

2.1 *The heteroscedasticity issue*

Model (1) assumes the homoscedasticity of the HEPs, but this assumption is not realistic for at least two causes described in literature (Knittel and Roberts, 2005; Koopman, Ooms, Carnero, 2007):

1 volatility clustering;
2 time-varying/periodic volatility.

These causes may be considered assuming the one step-ahead forecast error, $e_t = p_t - \widehat{p}_{t|t-1}$, comprising three factors:

$$e_t = u_t\sqrt{\exp(\lambda_t)h_t} \qquad u_t \sim i.i.d.(0,1)$$

where λ_t is a seasonal factor; h_t is a scale factor; u_t is a stochastic error, independent identically distributed with mean zero and variance one.

The crucial issue is how we model λ_t and h_t in order to define the squared volatility $\sigma_t^2 = exp(\lambda_t)h_t$. According to Koopman *et al.* (2007) we could model σ_t^2 by a periodic GARCH model similar to this one[1]:

$$\begin{aligned}
\sigma_t^2 &= \exp(\lambda_t)h_t \\
\lambda_t &= -\sum_{j=1}^{23}\lambda_{t-j} \\
h_t &= \omega + \sum_{j=1}^{h}\alpha_j \exp(-\lambda_{t-j})e_{t-j}^2 + \sum_{j=1}^{k}\beta_j h_{t-j}.
\end{aligned}$$

It is easy to note that λ_t is a periodic function depending on 23 parameters since λ_t can be rewritten as:

$$\lambda_t = \sum_{j=1}^{23}\gamma_j[d_j(t) - d_{24}(t)]$$

where $\gamma_j s$ are hourly parameters and $d_j(t)s$ are hourly dummies.

[1] Actually, the model of Koopman *et al.* (2007) includes regression effects too.

This approach allows us to estimate the ARIMA and GARCH models simultaneously, but the periodic GARCH model requires many parameters, i.e., 23 for the seasonality alone.

An alternative approach might be to define both λ_t and h_t as stochastic latent factors:

$$\lambda_t = -\sum_{j=1}^{23} \lambda_{t-j} + \varepsilon_{3,t} \qquad \varepsilon_{3,t} \sim n.i.d.(0, \sigma_3^2)$$

$$\ln h_t = \alpha + \sum_{j=1}^{k} \beta_j \ln h_{t-j} + \varepsilon_{4,t} \qquad \varepsilon_{4,t} \sim n.i.d.(0, \sigma_4^2)$$

Set $x_t = \ln h_t$, we can define the following structural time series model:

$$\begin{aligned} \ln e_t^2 &= \kappa_1 + \lambda_t + x_t + \eta_t && \eta_t \sim i.i.d.(0, \kappa_2) \\ \lambda_t &= -\sum_{j=1}^{23} \lambda_{t-j} + \varepsilon_{3,t} && \varepsilon_{3,t} \sim n.i.d.(0, \sigma_3^2) \\ x_t &= \alpha + \sum_{j=1}^{k} \beta_j x_{t-j} + \varepsilon_{4,t} && \varepsilon_{4,t} \sim n.i.d.(0, \sigma_4^2) \end{aligned} \qquad (2)$$

where $\kappa_1 = E[\ln u_t^2]$, $\eta_t = \ln u_t^2 - \kappa_1$, and $\kappa_2 = Var[\ln u_t^2]$.

The values of κ_1 and κ_2 depend on the distribution of u_t: in case of standard normal distribution, $\kappa_1 = -1.27$ and $\kappa_2 = 4.93$; in case of standardised Student's t-distribution, $t_v\sqrt{(v-2)/v}$:

$$\kappa_1 = -1.27 - \psi^0(v/2) + \ln \frac{v-2}{2}$$

$$\kappa_2 = 4.93 + \psi^1(v/2)$$

where v are the degrees of freedom of the t-distribution, ψ^0 and ψ^1 are the *digamma* and *trigamma* functions respectively (Abramowitz and Stegun, 1970).

The proposed model can be viewed as a seasonal version of the classic stochastic volatility (SV) model (Harvey, Ruiz and Shepard, 1994; Shepard, 2005), which was originally referred to as the *log-normal autoregressive* model by Taylor because the logarithm of the volatility follows an autoregressive model (Taylor, 1982).

Since model (2) is formulated in the state space form, forecasts and parameters estimates can be obtained using a Kalman filter, although η_t is not Gaussian, and the estimation method is the quasi-maximum likelihood (QML) method in this case (Ruiz, 1994; Shepard, 2005).

Now, the unknown parameters are only α, β, σ_3^2, and σ_4^2, since the seasonality depends only on a variability parameter (σ_4^2) rather than 23 hourly parameters.

Then, the squared volatility forecasts are derivable from the forecasts of the structural components, λ_t and x_t:

$$\begin{aligned} \widehat{\sigma}_{t+1}^2 &= \exp(\widehat{\lambda}_{t+1})\widehat{h}_{t+1} \\ &= \exp(\widehat{\lambda}_{t+1} + \widehat{x}_{t+1}) \end{aligned}$$

2.2 *Forecasting vs filtering*

As stated in Introduction, the day ahead electricity markets clear the 24 HEPs of the next day together. Then, models (1) and (2) cannot be used to predict each next price p_{t+1} knowing always the current price p_t as if the market were a continuous market. Obviously, we can use these models to make forecasts one, two,..., twenty-four hours ahead, but the forecasts of the late day hours (twenty or more hours ahead) will be always less accurate than the early day hours (few hours ahead). Actually, models (1) and (2) should be principally thought and used as filtering models rather than forecasting models: their strength is they allow to draw seasonality from HEPs and from hourly volatilities. That is useful to better understand the seasonality into prices and volatilities, and than to shape more accurate forecasting models. For example, we might obtain interesting one day ahead hourly forecasts combining the last seasonality estimates with the forecast of the daily price (average price): $\hat{p}_{h,d+1} = \hat{s}_h + \hat{p}_{d+1}$. Moreover, we might shape the hourly effects and the price level (after their filtering) by suitable long term models in order to have long term hourly price forecasts.

3. ANALYSIS OF THE ITALIAN PUN

We analysed the Italian hourly price for electricity (PUN) during the period from January 1, 2014 to April 30, 2014 (120 days, 2880 hourly prices), where Figure 1 shows the time series plot. This graph shows 2880 hourly prices, and some of the features mentioned in the first section can be observed, such as volatility clustering, mean reversion, and jumps and spikes, whereas seasonality is not clearly evident.

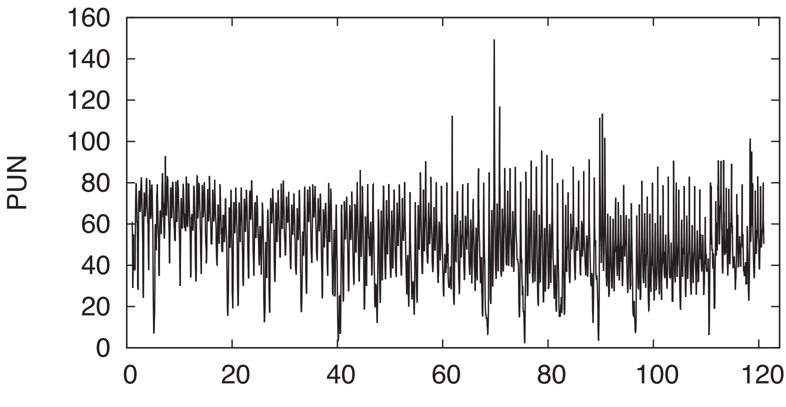

FIGURE 1. - *Hourly PUN, Jan. 2014-Apr. 2014*

The presence of seasonality is more evident in the plots of the means per hour and the standard deviations per hour of the PUN (Figure 2).

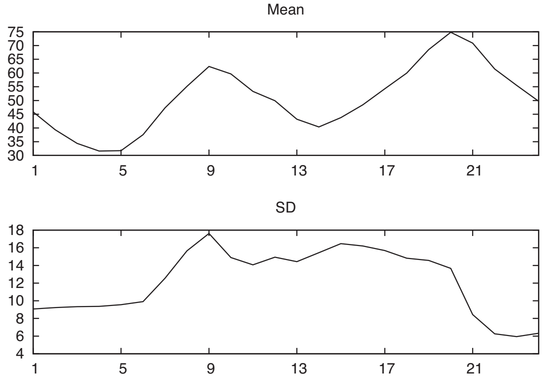

FIGURE 2. - *PUN's means and standard deviations per hour*

For the hourly means, we note that there is a ‘‘roller-coaster’’ pattern where two periods have relatively high prices, i.e., the morning and the evening, and the following periods are characterised by lower prices, i.e., the early afternoon and night/early morning. This pattern is consistent with the demand for electricity (due to human activities) during the day. However, the hourly standard deviation follows a different pattern, which is high from 8.00 a.m. to 8.00 p.m., before decreasing until 10.00 p.m. and then remaining low until 6.00 a.m.

The standard deviation is positively correlated with the mean only during the early hours of the morning and the last hour of the evening, but it is not proportional to the level. Thus, logarithmic transformation does not appear to be necessary to make the volatility stationary, which is an interesting difference from most financial prices.

According to these findings, the price level was shaped by the autoregressive level model with seasonal effects (1); Table 1 reports the parameters estimates.

About the variability of the disturbances, we note: σ_0 is not significant, that means almost every shocks, included spikes and jumps, produce some effect on the next hourly price; the hourly effects are stochastic, but slowly variable, since σ_2 is significant, but quite low.

The autoregressive level is characterized by the lags 1, 3, 4, 5, 6, 12 and 24 (the sum of the autoregressive coefficients is $0.794 < 1$). This model represents quite well the electricity prices as proved by the correlogram for the one step-ahead forecast errors (Figure 3).

The presence of seasonality in the standard deviations and the presence of volatility clustering also highlight the need for a model of volatility. According to the considerations discussed in the previous section, the errors e_t were treated by an SV model with seasonal effects similar to (2); more specifically a SV(1) with seasonal effects and standardised t_{10} errors[2] (Table 2).

[2] The order model and the degrees of freedom of Student's-*t* distribution were chosen on the basis of the information criteria of Akaike and Schwarz).

TABLE 1. - *Estimates of model (1)*

param.	value	std.err	z	p-value	sign.
σ_0	0.000	0.158	0.000	0.999	
σ_1	3.827	0.067	57.12	0.000	***
σ_2	0.504	0.031	16.26	0.000	***
δ	10.503	0.390	26.931	0.000	***
ϕ_1	0.949	0.013	70.460	0.000	***
ϕ_3	-0.108	0.022	-4.831	0.000	***
ϕ_4	-0.062	0.026	-2.421	0.016	**
ϕ_5	0.083	0.026	3.218	0.001	***
ϕ_6	0.041	0.019	2.171	0.030	**
ϕ_{12}	-0.039	0.009	-4.230	0.000	***
ϕ_{24}	-0.070	0.008	-8.397	0.000	***
Log-likelihood		-8369.6	Akaike		16759.2
Schwarz		16818.7	Hannan-Quinn		16780.7

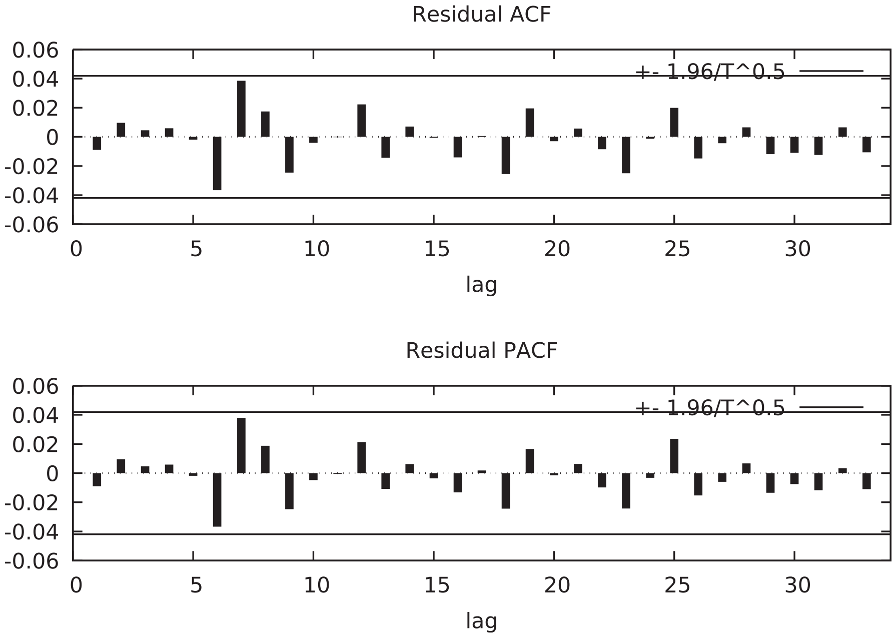

FIGURE 3. - *Correlogram for the forecast errors*

TABLE 2. - *Estimates of model (2)*

param.	value	std. err	z	p-value	sign.
σ_3	0.230	0.035	6.498	0.000	***
σ_4	0.077	0.014	5.548	0.000	***
α	0.100	0.030	3.328	0.001	***
β	0.960	0.012	82.56	0.000	***
Log-likelihood		-6299.510	Akaike		12607.02
Schwarz		12630.64	Hannan-Quinn		12615.56

We can note that all model parameters are significant, particularly σ_4, that means the hourly effects on volatility are not periodically constant; β is slightly less than one, that proves the persistence in volatility is high.

3.1 *Model validation*

If both models (1) and (2) are well specified, the (estimated) standardised forecast errors:

$$\widehat{u}_t = \frac{e_t}{\exp(\widehat{\lambda}_t/2 + \widehat{x}_t/2)} \qquad (3)$$

should have a standardised Student's t_{10}-distribution. As we can note in Figure 4 reporting the Q-Q plot of $\widehat{u}_t$s versus standardised t_{10}, the standardised forecast errors seem to respect the assumption (std. t-distribution) except the last three errors which are related to some particularly large spikes. Actually, the first errors seems to deviate from the assumption too, but slightly. Nevertheless, all the outliers are less then 0.3% of the data, and then the volatility predicted by the Kalman filter in the SV model (1) appeared to be consistent with the model's assumptions.

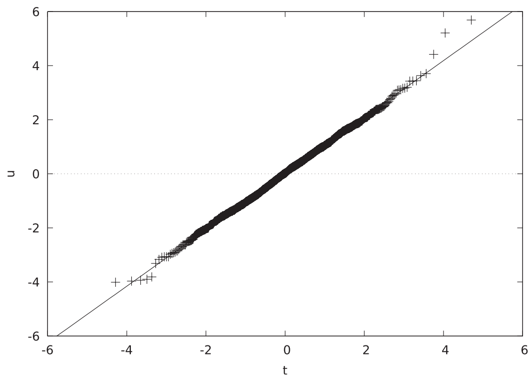

FIGURE 4. - *Q-Q plot of $\widehat{u}_t$s vs std. t_{10}*

4. CONCLUSIONS

Hourly electricity prices (HEPs) are characterised by strong intra-day seasonality, which affects the level and volatility of prices, and many parameters would be required for ARMA-GARCH modelling.

In this study, a new modelling approach was proposed, which employs structural time series modelling for both the level and volatility of the price. In both cases, seasonality is treated as a latent stochastic component that requires few parameters.

The strengh of this modelling is the capability of filtering the hourly effects from the level and volatility of HEPs, that should be a preliminary step towards shaping suitable forecasting models. The modelling was applied to the Italian PUN and significant results were obtained, although this approach has the following weaknesses: (i) the SV model is estimated by a QML method, which is a sub-optimal method; (ii) the level and volatility models are estimated separately, whereas conjoint estimation would be better. However, the first weakness is less significant than it appears because the experiments reported by Ruiz (1994) suggest that his QML method works rather well for the series that are typically encountered in financial economics. The conjoint estimation of both models would be more complex, but we aim to address this issue in our future research.

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