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A COMBINED CLUSTERING AND MULTI-CRITERIA APPROACH FOR PORTFOLIO SELECTION

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SUMMARY

One of the key role of a portfolio manager is to identify a suitable asset allocation strategy. The portfolio composition task has to take into account the return and the risk of each asset along with several other factors such as investor's aims, market expectations and risk tolerance. Thus the final decision making can be naturally viewed as a multiple criteria problem whose solution can benefit from Multiple Criteria Decision Making strategies. In this paper, we propose a two-step multi-criteria approach to support the selection of equity portfolios. Our strategy exploits a varying-coefficient Capital Asset Pricing Model framework. First, we identify clusters of stocks having similar systematic risk factors, then we rank these assets using an ELECTRE III method. We also present a real data example on stocks of S&P500 to illustrate the proposed methodology.

Keywords: *MCDM, P-Spline, Time Series, CAPM, Time-Varying Beta.*
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1. INTRODUCTION

Asset management is crucial for the definition of investment strategies. Due to the importance of portfolio selection in project and engineering management, many proposals have been made since the pioneer work of Markowitz (1952) suggested a mean-variance (MV) asset allocation framework.

This model assumes that, over a single period, a rational investor maximizes the expected return of its investments and minimizes the associated risks, measured by the variance of the assets returns. Hence, assets covariances reflect the importance of diversification in mitigating selection risks.

Starting from Markowitz results, Tobin (1958) studied optimal funds allocation, when cash or risk-free assets are considered as alternative to the risky ones.

The Capital Asset Pricing Model (CAPM) proposed by Sharpe (1964) and inde-

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pendently developed by Lintner (1965) and Mossin (1966), theorizes a linear relationship between stock returns and market-specific dynamics, measured by a factor, known as beta. Beta measures how much the stock price moves in line with the market. The CAPM explains that the expected return of an asset is comprised of a risk-free rate and a return associated with a market premium. It represents the risk associated with movements in the overall market. Since it is affected by all the assets, the market premium cannot be diversified. The securities moving closely with the excess market risk will have higher returns then assets that do not. Under the (unrealistic) CAPM hypothesis, the beta coefficients are fixed over time. Groenewold and Fraser (1999) showed that the CAPM beta is not stable over time. A more suitable model should consider a time varying component (see e.g. Valle, Meade and Beasley, 2015). The static CAPM is not able to capture differences in the behavior of an asset during different period producing scenario invalidating the classical theory. The results showed by Fama and French (1992) and Jagannathan and Wang (1996), highlight that the asset-specific betas and their returns strictly depend on (among the others) the information available at the given time about the financial markets and the overall economic conditions. Steuer and Na (2003) and more recently Zopounidis, Galariotis, Doumpos, Sarri and Andriosopoulos (2015) have recognized that, given all the constraints of the asset allocation process, portfolio selection problems cast within a Multi-Criteria Decision Making (MCDM) framework (Xidonas, Mavrotas and Psarras, 2010).

Multi-criteria classification is mainly focused on the assignment of actions to predefined classes. Decision makers have to group alternatives into classes by considering numerous conflicting criteria. Within a financial multi-criteria context, the portfolio selection problem is a typical example of decision making problem (Zopounidis, 1999).

Applications of MCDM methodologies in portfolio selection and financial decision making have been investigated by Zopounidis and Doumpos (2002), Hallerbach and Spronk (2002) and Spronk, Steuer and Zouponidis (2016). According to Spronk *et al.* (2016), MCDM methods have three main areas in a financial framework. They are the capital budgeting (how to evaluate and to choose between competing real investment projects), the capital structure (how the firm provides money to maintaining adequate level of solvency and liquidity) and financial investments. This latter consists of building a portfolio of securities according to the preference of the investors and it is made up of two steps, that is the choice of securities after the evaluation of the available stocks and the specification of the amount of capital to be invested. In a decision making process the aggregation of all the evaluation criteria is a crucial step. Within the MCDM problem, this phase can be carried out by using different models (outranking relations, utility function or decision rules). To support financial operators in the strategic asset allocation process, Beraldi, Violi and Simone (2011) proposed a decision support system which integrates mathematical modeling with simulation and optimization techniques. Ho, Tsai, Tzeng and Fang (2011) adopted a MCDM model to select a portfolio based on the Capital Asset Pricing Model (CAPM), while Lourenço, Morton and Bana e Costa (2012) presented a new decision support system to evaluate the robustness of a portfolio. Aouni, Colapinto and La Torre (2014) used goal programming enabling a decision maker to incorporate the various constraints and goals in asset allocation. For portfolio allocation purposes, Alali and Tolga (2019) adopted a multi-criteria decision making tool.

Statistical and machine learning techniques are also available for portfolio selection. Silva and Marques (2010) proposed an approach based on feature clustering with Self-Organizing Map (SOM). Nanda, Mahanty and Tiwari (2010) builds efficient portfolios by clustering stock returns and then selecting the optimal assets combination from the resulting groups. Menardi and Lisi (2015) adopted a three-step classification scheme based on a double clustering analysis to rank different mutual funds in terms of risk and profitability profiles. Zhang and Maringer (2009) proposed to combine a clustering technique with asset allocation methods. Cluster-based portfolio tend to show improved Sharpe ratios and weights stability (see, e.g. Zhang and Maringer, 2009). Tola, Lillo, Gallegati and Mantegna (2008) shows that portfolios based on clustering selection appears more reliable in terms of predicted vs. realized risk ratio. More recently, to support the investment decision of investors a P-spline based clustering approach for portfolio selection was introduced by Iorio, Frasso, D'Ambrosio and Siciliano (2018). A mixed-integer linear programming model for facing the clustering and asset selection problem in a unified framework was introduced by Puerto, Rodriguez-Madrena and Scozzari (2020). Caçador, Dias and Godinho (2020) defined a method for computing relative-robust and absolute-robust minimum variance portfolios. In a non-parametric estimation framework, Pandolfo, Iorio, Siciliano and D'Ambrosio (2019) proposed the application of statistical weighted depth functions for the construction of a robust mean-variance portfolio.

In this work, we propose a multi-criteria approach for selecting attractive securities to be included in investment portfolios. Hence, our risk evaluations are guided by a preliminary cluster analysis of time varying asset-specific beta coefficients. Our decision making model is intended to provide a tool that can aid the financial practitioners to select the stocks in accordance with the systemic risk factors, and not to build "the best portfolio", since the choice of which type of financial portfolio should be built involves a trade-off between rationality (subjectivity) and objectivity of the individual investor. By combining a clustering and a multi-criteria approach can be useful in the process of a financial decision making. In particular, our proposal may bring a decrease in the portfolio systemic risk required by an investor.

The paper is organized as follows. Section 2 reviews clustering and multi-criteria approaches. Section 3 presents our proposal. In Section 4 we evaluate the performances of our methodology by analyzing a large set of financial time series. The discussion in Section 5 concludes the paper.

2. THEORETICAL BACKGROUND

2.1 *Clustering*

Cluster analysis is a very popular tool in data analysis and finds application in several fields of science as engineering, computer science, medical science, social science and economics. In recent years, clustering techniques has become an increasingly important topic also for building and selecting portfolios. Clustering is an unsupervised learning problem and aims to partition a finite (unlabeled) data set into a finite and discrete set of natural hidden data structures based on its features.

Let $\mathbf{Z}$ be a data matrix nxp to be clustered where n represents the objects and p the variables. Let i be the subscript for objects and K be an integer, with $2 \leq k < K$. Indicate with μ_{ik} the membership function and let $\boldsymbol{P} = [\mu_{ik}]$ be the $n \times K$ partition matrix.

Crisp clustering methods are based on classical set theory and require each observation to belong to exactly one cluster. This means partitioning the data $\mathbf{Z}$ into a specified number of mutually exclusive clusters $C_1, \ C_2, \ \ldots, \ C_K$ such that the k^{th} column of $\boldsymbol{P}$ contains the value of μ_{ik} of the k^{th} subset C_k.

A hard partition of $\mathbf{Z}$ can be defined as a family of subsets C_K satisfying the following properties:

$$\cup_{k-1}^{K} C_k = \mathbf{Z}, \ C_k \cap C_j = \varnothing \ \forall k \neq j, \ \varnothing \subset C_k \subset \mathbf{Z}, \ 1 \leq k < K.$$

The elements of $\boldsymbol{P}$ must satisfy the following conditions:

$$\mu_{ik} = \{0, 1\}, \quad 1 \leq k \leq K, \quad 1 \leq i \leq n,$$

$$\sum_{k=1}^{K} \mu_{ik} = 1, \qquad 0 < \sum_{i=1}^{n} \mu_{ik} < n.$$

Fuzzy clustering allows the objects to belong to several clusters simultaneously, with different degrees of membership. The larger the value of the membership value for a given object with respect to a cluster, the larger is the probability of the object to be assigned to the specific cluster.

Generalizing the crisp partition, $\boldsymbol{P}$ is a fuzzy partition of $\mathbf{Z}$ with elements μ_{ik} of the partition matrix bearing real values in $[0, 1]$ (Kaufman and Rousseeuw, 2009). Similarly to crisping conditions, the conditions for a fuzzy partitions (Ruspini, 1970) are:

$$\mu_{ik} = [0, 1], \qquad 1 \leq k \leq K, \qquad 1 \leq i \leq n,$$

$$\sum_{k=1}^{k} \mu_{ik} = 1, \qquad 0 < \sum_{i=1}^{n} \mu_{ik} < n.$$

Han, Pei and Kamber (2011) distinguishes between partitioning methods, hierarchical, density-based, grid-based and model-based methods. A partitioning algorithm allocates the observations into K (with $K < n$) groups satisfying the following requirements: (i) each group contain at least one object, and (ii) each object belongs to exactly one group. A partitioning method uses an iterative relocation technique moving objects from one group to another in order to optimize a given criterion.

In what follows, we adopt the k-means (partitioning) algorithm (MacQueen, 1967; Hartigan and Wong, 1979). k-means partitions a set of objects into K clusters maximizing the intra-cluster similarity while minimizing the inter-cluster one. Cluster similarity is measured by evaluating the distance of every object to the centroid of each cluster.

2.2 *Multi-criteria analysis*

Multi-criteria analysis is well-known as Multi Criteria Decision Making (MCDM) or Multi Criteria Decision Aid (MCDA), according to the American or European School, respectively (Zopounidis, 1999). Anyway, multi-criteria analysis comprises different methods aiding decision makers in the evaluation of alternative decisions (Roy, 2016).

Many MCDM methods can be found in the literature and various categorizations of them are possible (Triantaphyllou, 2013). One classification is based on the type of data used, distinguishing among deterministic, stochastic or fuzzy methods.

According to the fuzzy set theory developed by Zadeh (1965), Bellman and Zadeh (1970) and Zimmermann (1978, 1983) identified two main principles categories of MCDM problems, namely Multiple Attribute Decision Making (MADM) and Multiple Objective Decision Making (MODM). MADM concerns for the selection or ranking of a predetermined as well as limited number of decision alternatives, which are known from the beginning. MODM involves the selection of an infinite number of unknown decision alternatives, designed in a mathematical framework that represents the trade-offs within a set of objective and constraints functions. Given their specific characteristics, MADM and MODM are also known as discrete and continuous multi-criteria problems, respectively (Kahraman, 2008). One can refer to Ribeiro (1996) for a survey about fuzzy decision making.

Another distinction is based on the number of decision makers involved (single or group decision makers MCDM methods). It is also possible categorize the multitude of MCDM methods according to the type and feature of the information, depending it on the faculty of decision maker to require, or not, additional information.

The most used methods in literature are Weighted Sum Method (WSM), Weighted Product Method (WPM), Technique for Order Performance by Similarity to Ideal Solution (TOPSIS), Analytic Hierarchy Process (AHP), Elimination and Choice Translating Reality (ELECTRE) and Multi Attribute Utility Theory (MAUT). All of them are popular discrete evaluation techniques. We refer to Chen and Hwang (1992) for a complete taxonomy of multi criteria analysis.

In this work, we will consider the ELECTRE method proposed by Roy (1968). This approach is based on binary relations defined on a set of alternatives, and the outranking relations require the definition of the concordance and discordance conditions as well as the notion of preference and indifference (Figueira, Mousseau and Roy, 2005). During the years, different versions of ELECTRE have been proposed (Roy, 1991; Wang J.Q., Wang D.D., Zhang and Chen, 2014) and they all consist of two steps:

1. first the outranking relations are constructed and
2. then the procedures of choosing, selection, sorting o ranking among the alternatives are applied.

Ranking problems are faced by the ELECTRE II, ELECTRE III and ELECTRE IV versions while ELECTRE Tri is used for sorting problems. Martel, Khoury and Bergeron (1988) used ELECTRE I and ELECTRE II to compare in an alternative way the financial portfolio, ranking a portfolio of securities based on logarithmic variance, firm capitalization, price/earning ratio and expected monthly returns. The ELECTRE III and ELECTRE Tri methods was employed by Xidonas, Askounis and Psarras (2009) in first two steps of stock classification and ranking of the methodology proposed to build common stock portfolios. Moreover, the authors also used ELECTRE III in the last stage of their proposal to ranking the efficient portfolio from a third step dealing with the optimization phase according MV criterion.

3. TIME-VARYING BETA CLUSTERING

We propose a two-step procedure combining hard clustering of risk factors and ELECTRE III ranking procedure, for the selection of investment portfolios. Our aim is to select, within a large set of alternatives, suitable assets to compose an investment portfolio by appropriately evaluating the associated systematic risk. We propose to cluster the beta coefficients for a set of assets. We model the risk indicators by means of P-spline whose coefficients are clustered so that each group is characterized by stocks with similar systemic risk profiles. Then we compute a series of relevant performance measures for each stock. Finally, these indexes are used as input of the ELECTRE III method in order to obtain a stock ranking useful for the asset selection step. Then a portfolio manager can select the $n < N$ top stocks according to ranking, given the previous screening clustering based on different profiles of systemic risk ensuring a better diversification of portfolio.

3.1 *P-splines basics*

P-splines (Eilers and Marx, 1996) are flexible smoothing procedures combining B-splines (de Boor, 1978) and difference penalties. Suppose to observe a set of data $\{x, y\}_{j=1}^{n_i}$, where x and y are vectors indicating the independent variable (e.g. time) and the dependent one (e.g. securities return), respectively. We want to describe the available measurements through an appropriate smooth function. Denote $B_h(x; q)$ the value of the hth B-spline of degree q defined on a domain spanned by m equidistant knots (in case of not equally spaced knots our reasoning can be generalized using divided differences). A curve that fits the data is given by $\hat{y}(x) = \sum_h a_h B_h(x; q)$ where a_h (with $h = 1, \ldots, m + q$) are the estimated B-splines coefficients. Unfortunately the curve obtained by minimizing $||y - Ba||^2$ w.r.t. a

shows more variation than is justified by the data if a dense set of spline functions is used. To avoid over-fitting, a can be estimated within a penalized regression framework:

$$\hat{a} = arg\ \min_{a} ||y - Ba||^2 + \lambda||D_d a||^2, \tag{1}$$

where D is a dth order difference penalty matrix and λ is a smoothing parameter. Second or third order difference penalties are suitable in many applications. The optimal spline coefficients follow from (1) as:

$$\hat{a} = \left(B' B + \lambda D_d' D_d\right)^{-1} B' y.$$

For a complete discussion about the properties of the P-spline smoother one can refer Eilers and Marx (1996, 2010).

The smoothing parameter λ controls the trade-off between smoothness and goodness of fit. For $\lambda \to \infty$ the final estimates tend to be constant while for $\lambda \to 0$ the smoother tends to interpolate the observations. Popular methods for smoothing parameter selection are: the Akaike Information Criterion (AIC), the Schwartz Bayesian Information criterion (BIC), which will be used in the applications presented in this paper), the generalized cross validation. Most recently, another selection procedure, derived from the L-curve (Hansen, 1992) was proposed by Frasso and Eilers (2015).

3.2 *Time-varying coefficients model using P-Splines*

When investors make portfolio choices, the riskiness of the selected assets needs to be modeled. Within the CAPM framework one can distinguish between specific and systemic risk factors. The first is associated with specific asset and can be diversified, while the latter refers to the risk which is not diversifiable, common to all the assets and measured by the parameter β. It is a measure of an asset's systematic risk. Within a static CAPM framework, the following relationship holds:

$$r_i(t) = rf + \beta_i(rm(t) - rf), \tag{2}$$

where r_i is the return for asset i, rf is the risk-free rate (which is known), β_i is the sensitivity of the expected asset returns to the market returns rm (as measured by a stock market index for example).

Equation (2) assumes constant βs. A more flexible formulation allows the risk factors to vary over

$$r_i(t) = rf + \beta_i(t)(rm(t) - rf). \tag{3}$$

Equation (3) is a special case of varying-coefficient model (VCM, see e.g. Hastie and Tibshirani, 1993). We model $\beta_i(t)$ using P-spline (see e.g. Eilers and Marx, 2002). Equation (3) can then be formulated as

$$y_i(t) = a_{0,i} + diag\{x(t)\}Ba_{i,i} + \varepsilon_i(t) = (\mathbb{1}|U)\alpha_i + \varepsilon_i(t) = Q\alpha_i + \varepsilon_i(t),$$

where $\varepsilon_i(t)$ is a zero mean error term with constant variance, $y_i(t) = r_i(t) - rf$, $x(t) = rm(t) - rf$, $\alpha_i = \left(a'_{0,i}, a'_{1,i}\right)$, $a_{0,i}$ is an asset-specific intercept term, $a_{1,i}$ is the vector of spline coefficients for the time-varying risk factor for asset i, B is a B-spline matrix and $U = diag\{x(t)\}B$ with $diag\{x(t)\}$ aligning the predictors with the appropriate smooth slope values. If $Q = (\mathbb{1}|U)$, then the penalized estimation problem for asset i becomes:

$$S_i = ||y_i(t) - Q\ a_i||^2 + \lambda_i||\breve{D}_d a_i||^2, \tag{4}$$

where $\breve{D}_d$ shrinks only the $\alpha_{1,i}$ coefficients in α_i and λ_i is a smoothing parameter. The solution of (4) is then

$$\hat{a}_i = (Q'Q + \lambda_i \breve{D}'_d \breve{D}_d)\ ^1 Q' y_i(t), \tag{5}$$

from which it follows that

$$\hat{\beta}_i(t) = B\hat{a}_{1,i}.$$

3.3 *K-means clustering of time-varying β*

In analogy with Iorio *et al.* (2016), we group the asset-specific time-varying βs (estimated as in Section 3) using a k-means procedure. In a second step we compute risk-adjusted performance measures for each security belonging to the different cluster. We use these indicators as criteria of ELECTRE III method to rank the assets for each resulting group.

The k-means algorithm partitions the observations in a number K of groups chosen a priori. We select the optimal number of clusters using the GAP statistics proposed by Tibshirani, Walther and Hastie (2001). We compute (5) for each observed return series and store the $\hat{a}_{1,i}$ coefficients in the $\boldsymbol{BS}$ matrix of dimension $[(m+p) \times N]$. Then we assign randomly the columns of $\boldsymbol{BS}$ to K groups to form the initial clusters. When all objects have been assigned to a group, the positions of the cluster centroid are updated. The partitioning procedure is stopped when the cluster centers stabilize. The K centroids $(c_k | k = 1, ..., K)$ minimize the following equation:

$$\sum_{i=1}^{N} \sum_{k=1}^{K} \mathcal{D}(a_{1,i}, c_k),$$

where $\mathcal{D}(.)$ is a distance (dissimilarity) measure between the ith beta spline coefficient vector and the kth cluster center. In our applications, we used a metrics based on the Pearson's correlation coefficient:

$$\mathcal{D}_\rho = 1 - \rho\left(a_{1,i}, c_k\right),$$

where $\rho(.)$ indicates the Pearson's correlation coefficient.

4. REAL DATA APPLICATION

We test our procedure selecting a set of 48 randomly selected stocks included in the S&P500 Index. The data were acquired from *yahoo.finance.com* on monthly basis from January 2006 to December 2010. For each closing price we computed the returns as log-price differences. Each series of monthly returns has been modeled by P-splines taking cubic bases defined over 30 equally spaced interior knots and third-order difference penalties. The optimal number of clusters was selected through a GAP statistics approach with 100 bootstrap replicates (here the 1-se rule suggests 3 clusters, see Table 1). Finally, in order to avoid local minima, we replicated the partitioning procedures 100 times.

TABLE 1. - *Table of the GAP statistics computed over 100 bootstrap samples*

K	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
GAP_K	0.01	0,07	0.11	0.14	0.14	0.17	0.20	0.21	0.24
sim S. E.	0.03	0.03	0.04	0.03	0.03	0.03	0.03	0.03	0.03

Figure 1 summarizes our approach results. The recognized centers distinguish between different market risk dynamics. We used the ELECTRE III approach to rank the securities belonging to each systematic risk dynamics group. We considered a set of 6 performance measures:
- Returns means
- Returns volatility (standard deviations)
- Adjusted Sharpe Ratio
- Conditional Value-at-Risk (CVaR)
- Upside potential ratio (UPR)
- Burke ratio drawdowns measure.

The Adjusted Sharpe Ratio (AdjSR) incorporates a penalty factor for negative skewness and excess kurtosis (Pezier and White, 2006). This allows us to take into account third and fourth moment of the realized asset return distributions. The Conditional Value-at-Risk (CVaR) is a coherent risk measure (Artzner, Delbaen Eber and Heath, 1999; Acerbi, 2002; Rockafellar and Uryasev, 2000, 2002) and is best known as Expected Shortfall (ES). ES describes the expected loss under the condition that VaR is exceeded, considering in its computation the values of the distribution that exceed the VaR only. The Upside potential ratio (UPR) (developed by Sortino, Van Der Meer and Plantiga, 1999) is obtained as ratio between upside potential and downside risk. More specifically, the numerator is the average sum of stock returns that exceed the Minimum Acceptable Return (MAR), while the denominator is the average sum of returns below the MAR. The drawdowns are measured as the maximum decrease in the value of the returns over a specific period of time. Among this category we computed the Burke ratio (Burke, 1994). This index divides the excess return of a portfolio above the risk free rate by the square root of the sum of the square of the drawdowns.

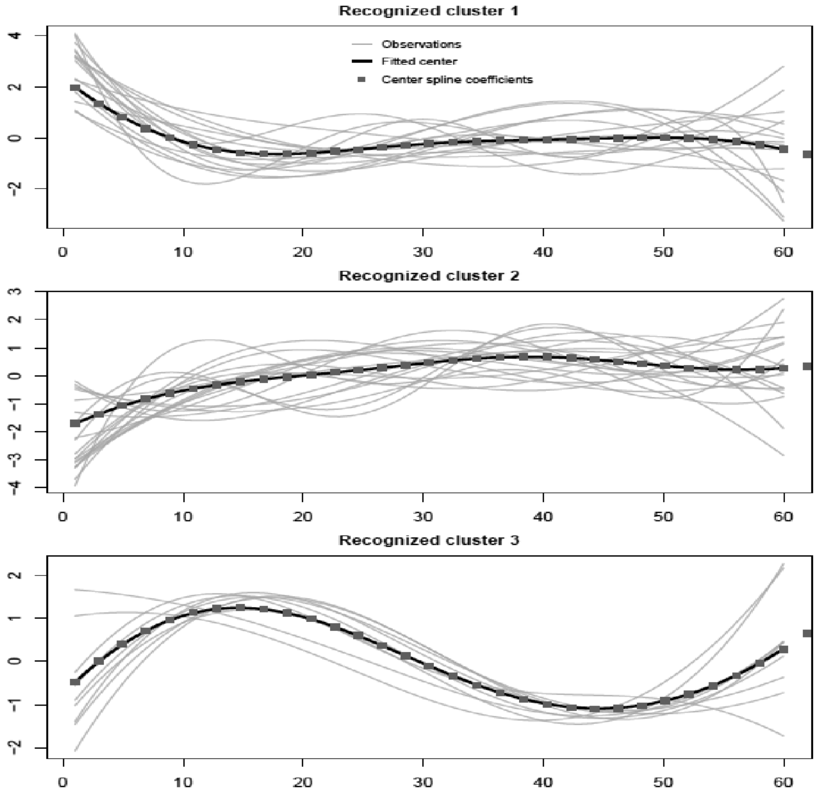

FIGURE 1. - *Beta varying coefficient clustered by k-means procedure with Pearson's correlation based distance. For each subplot the horizontal axis represents the time and the vertical axis the systematic risk level. The gray lines reproduce the beta series assigned to each cluster. The dots indicate the optimal P-spline coefficients for cluster center. The black solid lines indicate the center functionals*

Finally, in order to avoid possible double counting issues (occurring when two or more chosen performance measure are similar so that the respective weights lead to biased rankings) we checked for the linear dependency. We computed among the selected performance measure their correlation for each cluster and we did not find any evidence for double counting issues. In ELECTRE III method each considered criterion requires as inputs indifference thresholds (q), preference thresholds (p) and veto thresholds (v). Since our aim is propose a strategy of stock ranking according to different group of systematic risk indicators we have set the indifference threshold constant for each criterion performance criterion. Table 2 shows the adopted thresholds (set experimentally).

TABLE 2. - *ELECTRE III thresholds*

	q	*p*	*v*
Mean	0.05	0.20	0.40
SD	0.05	0.10	0.30
AdjSR	0.05	0.20	0.40
ES	0.05	0.10	0.30
UPR	0.05	0.20	0.30
BR	0.05	0.10	0.10

Tables 3, 4, 5 present the ranking results in each cluster resulting from the previous *k*-means step. Then, starting from the obtained ranking, one can choose a set of $n < N$ securities belonging to the different groups of systematic risk.

TABLE 3. - *Ranking of stocks from cluster 1*

ELECTRE III	**Rank**	**ELECTRE III**	**Rank**
AAPL	1	ADI	5
AEP	2	ADBE	6
AGN	2	AIZ	6
ADT	3	AIG	6
AFL	3	AIV	6
ALLE	3	AON	6
APH	3	ALXN	6
AEE	4	APD	7

TABLE 4. - *Ranking of stocks from cluster 2*

ELECTRE III	Rank	ELECTRE III	Rank
AKAM	1	ADSK	4
A	2	AET	4
ABBV	3	AMGN	4
ABC	3	APC	4
ADM	3	AMP	5
ARG	3	MO	5
GOOG	3	AAP	5
AES	4	ALL	6
AMZN	4		

TABLE 5. - *Ranking of stocks from cluster 3*

ELECTRE III	Rank	ELECTRE III	Rank
AME	1	ACN	5
AMT	2	T	5
MMM	2	AA	5
GAS	3	AAL	5
ADS	4	APA	5
AMG	4	ANTM	6
ATVI	4	AMAT	7
ABT	5		

5. CONCLUDING REMARKS

In this paper we propose a portfolio composition method within a multi-criteria framework. This approach takes into account different levels of systematic risk as measured within a time-varying CAPM model settings. The procedure is based on a cluster analysis of the time-varying market risk factors to identify groups having homogeneous beta dynamics. Then, a set of performance measures is used as criteria for the ELECTRE III ranking method. This combination of clustering and multi-criteria can be a valid and helpful aid for financial practitioners when dealing with a portfolio selection problem. Even if the criterion chosen depend on the preferences and risk-aversion profile of a portfolio manager (or an investor), we believe that the combination of clustering and multi-criteria ranking procedures represent a valuable aid to the portfolio composition process. We have presented a case study with a real data set regarding the returns of 48 stocks included in the S&P500 for the period

2006-2010. It is important to stress that we are proposing a possible alternative strategy to build a financial portfolio and not to identify the best portfolio. Hence, any performance measures can be adopted after the clustering step is done.

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