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INCOME DISTRIBUTION AND INCOME INEQUALITIES IN POLISH HOUSEHOLDS

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SUMMARY

This paper aims to assess the direction of changes in the distribution of per capita income as well as income inequalities in Polish households. The main trends of change were distinguished on the basis of theoretical distributions estimated for 2009-2018. However, a detailed comparative analysis of income distribution and the decomposition of income inequalities by major socio-economic groups was carried out for 2015 and 2018 (before and during the implementation of significant social programmes). In the article we used methods of modelling distribution of per capita income, including the Dagum distribution, Bhattacharyya distance, Gini coefficient and Theil index. All analyses were performed on the basis of microdata from the Household Budget Survey of 2009-2018, carried out by Statistics Poland. The data set for each period consists in the observation of a minimum of 35 thousand households. The results of research indicate an increase in the level of per capita income in the period 2009-2018. The introduced changes in social policy have both resulted in an increase in the average level of income and a decrease in income inequality, especially in families with dependent children.

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1. INTRODUCTION

Income and the related inequalities constitute a starting point in the scope of research of the broadly-understood economic situation of entities and households. The results of analyses in this area are also the basis for conducting social policy. A part of the programmes realised under the aforementioned policy, especially those based on social transfer programmes, may significantly influence the level of obtained income, as well as the level of inequalities between individual persons or households. In Poland, *Rodzina 500+* (Family 500+) is considered to be one of such programmes, as part of which since April of 2016 every family, regardless of income,

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is entitled to parental benefits in the amount of 500 PLN for every second and subsequent child. For families with low per capita income, this benefit was also granted for the first child. From July 2019, the programme covers all children in the family with no income criterion. The motive for writing this paper is the desire to verify whether and how the said programme and other social benefits implemented since 2016 in Poland have affected the level of income and income inequalities.

The aim of the study is to assess the direction of changes in the distribution of per capita income as well as income inequalities in Polish households. The main trends of change were distinguished on the basis of theoretical distributions estimated for 2009-2018. A detailed comparative analysis of income distribution and the decomposition of income inequalities by major socioeconomic groups was carried out for 2015 and 2018 (before and during the implementation of significant social programmes).

2. METHODOLOGY AND STATISTICAL DATA

In this article, the theoretical Dagum distribution was used as the model of the distribution of per capita incomes. The density function $f(y)$ and the cumulative distribution function $F(y)$ of this distribution may be written as follows:

$$f(y) = \frac{cbexp(-a)y^{-(b+1)}}{[1+\exp(-\mathrm{a})y^{-b}]^{c+1}} \tag{1}$$

$$F(y) = \frac{1}{[1+\exp(-\mathrm{a})\mathrm{y}^{-\mathrm{b}}]^{c}}, \tag{2}$$

where a, b, c are parameters which are most often estimated using MLE (Maximum Likelihood Estimation). The Dagum distribution is recognised in the literature on income research as one of the best income distribution models for large samples (Kleiber and Kotz, 2003; 221-222). Subsequently, the distribution was used in Poland, e.g.: by Kot, Malawski and Węgrzecki (2004), Łukasiewicz and Orłowski (2004), Jędrzejczak (2009), Jędrzejczak and Trzcińska (2018) and Ulman (2015). The goodness-of-fit measures A_1 and W were used. The former – Mortara index- was used among others by Zenga, Pasquazzi and Zenga Ma. (2012); the latter is given by the following formula: $W = \sum_{i=1}^{k} (w_i;\ w'_i)$, where w_i and w'_I represents empirical and theoretical frequencies, respectively.

Measures of distance (overlapping) of distributions over time and distributions of individual socioeconomic groups were calculated using the Bhattacharyya distance (Bhattacharyya, 1943). It is based on a comparison of the density function of two distributions according to the following formula:

$$\rho = \int \sqrt{f_1(y) * \mathrm{f}_2(\mathrm{y})}dy, \tag{3}$$

$$d_b(f_1, f_2) = -\ln \rho, \tag{4}$$

where $\rho \in [0, 1]$, and the Bhattacharyya distance takes values from the interval

$0 \leq d_b \leq \infty$. The value of zero means identity of distributions (their overlap), and the higher the value of this distance, the more the tested distributions differ.

To demonstrate the real level of income changes, the calculation used per capita income at fixed prices from 2018. In order to measure the level of income inequality and its decomposition between the separated groups, we used the Gini coefficient, the relative average deviation (RAD), widely known as the Pietra inequality index, and the Theil index. Pietra inequality index, also known as the relative average deviation (RAD), is expressed as follows (Pietra, 1915):

$$P = \frac{s_{(\bar{y})}}{2\bar{y}}, \tag{5}$$

where $\bar{y}$ is the arithmetic mean of y and $s_{(\bar{y})}$ is the mean absolute deviation of y from $\bar{y}$. The Theil index can be expressed with the formula:

$$T = \frac{1}{n}\sum_{i=1}^{n} \frac{y_i}{\bar{y}} \cdot \ln \frac{y_i}{\bar{y}}. \tag{6}$$

The Theil index can be additively decomposed, allowing the decomposition of overall inequality due to the selected subgroups (Shorrocks, 1980). Therefore, the Theil index can also be expressed as a weighted sum of inequalities within groups of households and inequalities between these groups:

$$T_G = \sum_{i=1}^{k} w_s T_s + T_B. \tag{7}$$

where: w_s – decomposition coefficient given by: $w_s = \frac{n_s \bar{y}_s}{n\bar{y}}$ (n_s –size of the s-th subgroup, $\bar{y}_s$ – mean for the s-th subgroup, T_s – Theil index determined for the s-th subgroup (intra-group), TB – Theil index calculated on the basis of the mean values of individual subgroups (inter-group), which takes the form: $T = \frac{1}{n}\sum_{i=1}^{s} \frac{\bar{y}_s}{\bar{y}} \cdot n_s \cdot ln \frac{\bar{y}_s}{\bar{y}}$.

The problem of decomposition of income inequality in Poland was discussed by inter alia Jędrzejczak (2010) and Zenga and Jędrzejczak (2020).

All analyses were performed on the basis of microdata from the Household Budget Survey of 2009-2018. This survey is carried out each year by Statistics Poland. The data set for each of the periods is the observation of a minimum of 35 thousand households.

3. RESULTS

The average value of per capita income in Polish households has been successively increasing over the period of 10 years[1] (see Figure 1). This increase was observed

[1] Location and inequality measures were calculated based on Dagum distribution. The goodness-of-fit measures showed very good fit - A_1 is not bigger than 0.05 and W not lower than 0.97.

both in nominal values and taking into account the size of inflation. The increase in average nominal income per capita in 2018 compared to 2009 amounted to approx. 52% and real income 33%. A particularly dynamic increase in income in Polish households has occurred in the last three years from 2016 to 2018.

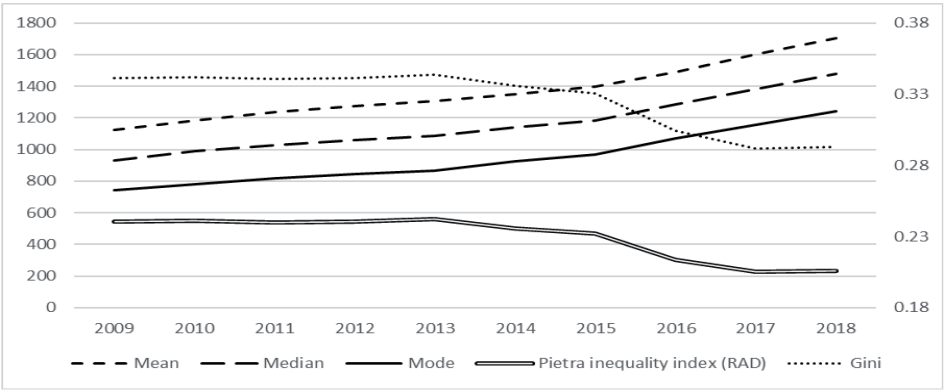

FIGURE 1. - *Location and inequality measures of income per capita in Polish households*

The level of inequality measured both by the Gini coefficient and the Pietra inequality index, indicates a significant decrease in these inequalities in the last part of the period under review. Considering these two circumstances, it can be stated that social wellbeing measured by the Sen welfare index[2] has increased distinctly in Poland.

Figure 2 presents the Dagum distribution density function of income per capita due to different characteristics of the household or the head of the household. The charts refer to 2018. Income distribution differs by socioeconomic group, class of place of residence, education level of household head and biological type of household. In the best financial situation are those households which belong to the group of the self-employed, living in big cities, whose head has tertiary education. Considering the biological type of household, in the best situation are households without dependent children. In the case of income distributions considered due to the aforementioned features of households, fitting the Dagum distribution to empirical distributions is slightly worse than in the case of the entire household population, although in most cases A_1 does not exceed 0.16 and W is not less than 0.9. Only in the case of marriages with a large number of dependent children, the abovementioned measures exceed said values, however, the Dagum distribution can still be considered as an appropriate model of income distribution for these households.

[2] Sen (1976) estabished that, giving some condition, the welfare level of a i-th country (Si) can be estimated by the formula: $S_i = \bar{y}_i \cdot (1 - G_i)$, where $\bar{y}_i$ denotes the mean income of i-th country and G_i is the Gini coefficient of the personal contribution in i-th country.

When examining the distance between income per capita distributions in the years 2013-2018, we can notice that the more distant the time of distribution, the greater their distance from each other (see Table 1). Knowing that in subsequent years the level of income per person in Poland increased, we can indicate the direction of this shift in income distribution - towards higher income values. The largest distances in income distribution from year to year occurred in 2018 compared to 2017.

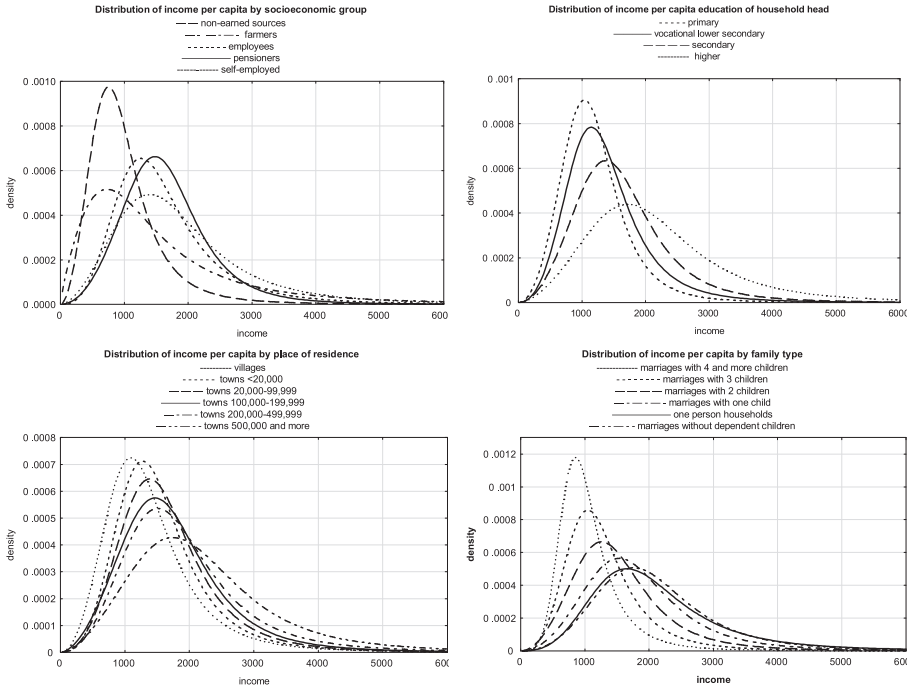

FIGURE 2. - *Distribution of income per capita in Polish households by socioeconomics group, education of household head, place of residence class and biological type of household*

TABLE 1. - *The Bhattacharyya distance of income per capita distributions by year of observation*

Year	2013	2014	2015	2016	2017	2018
2013	-	0.0207	0.0232	0.0348	0.0564	0.0694
2014		-	0.0212	0.0295	0.0476	0.0588
2015			-	0.0265	0.0416	0.0516
2016				-	0.0284	0.0362
2017					-	0.0312
2018						-

When comparing income distributions by socioeconomic groups, one can notice that the largest difference between income distributions is between households earning their living from self-employment and social benefits (see Table 2). The income distributions of households living on retirement and households obtaining money from social benefits are far apart from each other. This shows that income from pensions in Poland creates a much better financial situation than income from social benefits and is similar to the situation of households which earn their living from contract-based working relationships.

TABLE 2. - *The Bhattacharyya distance of income per capita distributions by type of family main source of income*

Main source of income	Year	Employment	Agriculture	Self-employment	Pension	Social benefits
Employment	2015	-	0.0808	0.0461	0.0257	0.1192
	2018	-	0.1197	0.0562	0.0259	0.1445
Agriculture	2015		-	0.1257	0.1010	0.0464
	2018		-	0.1271	0.1271	0.0920
Self-employment	2015			-	0.0386	0.1910
	2018			-	0.0491	0.1889
Pension	2015				-	0.1664
	2018				-	0.1733
Social benefits	2015					-
	2018					-
2015-2018	-	0.0566	0.0770	0.0745	0.0303	0.0397

Table 3 presents the Bhattacharyya distance for different household types in Poland. In 2015, the income distribution of marriages with four or more children was significantly distanced from the distribution of marriages without dependent children. In 2018 this distance was largely reduced. The same applies to marriages with fewer dependent children. The impact of this programme on the shaping of income distribution is also visible when compared in the two studied periods (bottom row). In the case of marriages with the largest number of children, these distributions are the most distant from each other; these households have gained the most from the mentioned social programme.

Table 4 demonstrates the Bhattacharyya distance depending on the class of the household's place of residence. In general, it can be concluded that the distance between distribution of households living in different localities increases with the difference in the size of the city. The per capita income distribution of households living in cities of 500,000 or more was considerably distanced from the distribution of households living in the village. These distances did not change significantly between 2015 and 2018.

TABLE 3. - *The Bhattacharyya distance of income per capita distributions by type of family*

Type of family	Year	Marriage without dependent children	Marriage with 1 child	Marriage with 2 children	Marriage with 3 children	Marriage with 4+ children	One person household
Marriage without dependent children	2015	-	0.0534	0.1277	0.2864	0.5617	0.0487
	2018	-	0.0574	0.0964	0.1735	0.3182	0.0666
Marriage with 1 child	2015		-	0.0457	0.1436	0.3445	0.0633
	2018		-	0.0449	0.0944	0.2062	0.0619
Marriage with 2 children	2015			-	0.0459	0.1792	0.1395
	2018			-	0.0311	0.0996	0.0956
Marriage with 3 children	2015				-	0.0545	0.3011
	2018				-	0.0345	0.1667
Marriage with 4 + children	2015					-	0.5820
	2018					-	0.3054
One person household	2015						-
	2018						-
2015-2018	-	0.0605	0.0487	0.0486	0.0856	0.1731	0.0701

TABLE 4. - *The Bhattacharyya distance of income per capita distributions by household place of residence*

Place of residence	Year	Villages	T <20,000	T 20,00-99,999	T 100,000-199,999	T 200,000-499,999	T >500,000
Villages	2015	-	0.0678	0.0795	0.0887	0.1132	0.1657
	2018	-	0.0783	0.0787	0.0893	0.1079	0.1417
T <20.000	2015		-	0.0263	0.0263	0.0396	0.0781
	2018		-	0.0403	0.0426	0.0522	0.0800
T 20,000-99,99	2015			-	0.0186	0.0252	0.0525
	2018			-	0.0265	0.0317	0.0515
T 100,000-199,999	2015				-	0.0170	0.0405
	2018				-	0.0222	0.0405
T 200,000-499,999	2015					-	0.0233
	2018					-	0.0278
T >500,000	2015						-
	2018						-
2015-2018	-	0.0906	0.0565	0.0461	0.0415	0.0469	0.0459

Further tables present the decomposition of per capita income inequality by socioeconomic group, size of family and household place of residence using the Theil index.

Although the value of the Theil index is the highest for farm households, the contribution of this social group to the creation of total inequalities is minor and amounts to slightly above 10%. This stems from the small number of such households. The largest part of income inequality per capita is generated by a group of employees (50.95% in 2015 and 50.08% in 2018) (see Table 5). At the same time, it represents the most numerous socio-economic group. The average per capita income in working households was very close to the total income. The least contribution into generating general inequalities was observed in households making a living on non-earned sources of subsistence. In 2015 these households were responsible for 4.4% and in 2018 for 3.8% of the total inequality. It is also the group with the lowest average income per person, whereby in 2015 the average income in this socio-economic group accounted for 68% of the average total income per capita.

TABLE 5. - *The Theil index of income per capita distributions by socioeconomic groups*

Socio-economics groups	2015			2018		
	N	Mean	Theil indeks	N	Mean	Theil indeks
Employees	18279	1631.23	0.1894	17102	1989.44	0.1595
Farmers	1509	1538.52	0.5648	1437	2006.75	0.4254
Self-employed	2445	2046.84	0.2541	2435	2383.20	0.2209
Retirees	13101	1548.61	0.1019	13621	1750.87	0.0883
Maintained from non-earned sources	1526	1087.15	0.2933	1239	1250.6718	0.2438
Total	36860	1603.11	0.1876	35834	1900.66	0.1591

When analyzing income inequalities by type of family, it should be noted that inequalities in general were most impacted by the differences between individual family types (see Table 6). In 2015, as much as 32% of the total inequality resulted from inter-group inequality and in 2018 this value decreased to 30%. Single-person households and married couples without children also had a conspicuous impact on inequality (24% in 2015 and 27% and 26% respectively in 2018). Families with three or more children and single parents (less than 2% in both analyzed years) had minimum influence on total income inequalities.

In the case of research on inequality due to households' place of residence, it should be noted that total inequality is most impacted by inequalities observed between households living in rural areas (36% in 2015 and 37% in 2018), which is caused, inter alia, by a large size of the aforementioned group (Table 7). The overall

inequality is also greatly influenced by inequalities between households in the largest cities (over 500,000 inhabitants). In both observed periods, these households generated 20% of total inequality. It is also the group with the highest average income per capita. As for the inequalities between groups, they decreased from 11% in 2015 to 9.5% in 2018. This is undoubtedly related to the reduction of inequalities between particular types of families, as usually people decide to have a smaller number of children in larger towns and villages.

TABLE 6. - *The Theil index of income per capita distributions by type of family*

Type of family	2015			2018		
	N	Mean	Theil indeks	N	Mean	Theil indeks
Marriage without dependent children	9312	1901.10	0.1507	9730	2151.61	0.1327
Marriage with 1 child	3817	1556.00	0.1500	3126	1898.03	0.1383
Marriage with 2 children	3964	1251.60	0.1953	3141	1580.47	0.1468
Marriage with 3 children	993	943.52	0.1679	715	1336.73	0.1391
Marriage with 4 or more children	289	667.48	0.1466	150	1099.84	0.1233
Single-person household	7406	2043.02	0.1779	7873	2244.02	0.1673
Marriage with at least 1 child and other peaple	2860	1069.52	0.1474	3391	1395.88	0.1463
Single mother/father	810	1184.44	0.1807	665	1495.49	0.1562
Others	7409	1377.84	0.1485	7043	1669.70	0.1354
Total	36860	1603.11	0.1876	35834	1900.66	0.1591

TABLE 7. - *The Theil index of income per capita distributions by place of residence*

Place of residence	2015			2018		
	N	Mean	Theil indeks	N	Mean	Theil indeks
T >500.000	4803	2362.87	0.1986	4536	2653.62	0.1762
T 200.000-499.999	3293	1832.04	0.1341	3016	2211.00	0.1311
T. 100.000 -199.999	2819	1722.98	0.1349	2899	2020.57	0.1093
T 20.000-99.99	6334	1613.34	0.1327	6293	1900.91	0.1106
T <20.000	4206	1489.62	0.1299	3867	1778.90	0.1158
Villages	15405	1322.14	0.1944	15224	1622.88	0.1640
Total	36860	1603.11	0.1876	35834	1900.66	0.1591

4. CONCLUSIONS

The research on income and distribution has a lengthy history. This stems from the significance of the issue both in the individual, as well as social dimensions. Income is an indicator of the well-being of an individual and of entire social groups. Thus, a comparison of the level of income in space and in time makes it possible to reach conclusions about the diversification of the aforementioned well-being and the effectiveness of social programs aimed at improving the living conditions of the economically weakest social groups, and thus minimizing economic inequalities.The results of research indicate an increase in the level of income per capita in Polish households in the period 2009-2018 both nationwide and in particular socioeconomic groups. The changes in social policy introduced in 2016 have resulted in an increase in the average level of income and a decrease in income inequality, which proves the effectiveness of the social policy in reducing income inequalities. The largest decrease in inequality was noted for marriages with dependent children. Such a considerable reduction of economic inequalities with a simultaneous growth of society's income basically affects the level of social welfare. From the theorems of Atkinson and Kolm in this respect, as well as from the abbreviated welfare function it results that welfare grows with an increase in income and/or a decrease in inequality in their distribution. Our research demonstrated that the last few years in Poland have seen conditions for a dynamic growth in social well-being measured in a simplified scheme.

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