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THE SAM'S GLOBAL MULTIPLIER MATRIX AS A ''STRUCTURAL'' INEQUALITY MEASURE OF PERSONAL INCOME DISTRIBUTION

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SUMMARY

Aim of this paper is to introduce the ''global multipliers matrix'' as a ''structural'' inequality measure of personal income distribution. This measure is derived from the Social Accounting Matrix, considered as a linear model. The multiplier approach allows quantifying the different ways by which, an income equally earned by each Household's group, turns into different disposable income levels through the three stages of spending, production and distribution. The resulting inequality in personal income distribution can be considered as the minimum inequality compatible with the given productive and spending structures, and hence as a result of the mechanism explicitly considered in the model. The multiplier matrix allows highlighting different features of personal income distribution and its inequality's level. In particular: i) the extent to which each sector of activity contribute to the distribution of value added (primary income) over the different household groups; ii) how the composition of production factors's ownership is linked to the intrahousehold inequality; iii) the impact of different fiscal and redistributive policies which translate into changes in disposable incomes of different household groups. This approach, we argue, allows for the assessment and evaluation of the effects of ''new policies'', aimed at reducing poverty and inequality ex-ante and not only ex-post. Some numerical example, referred to the Italian economy, allow quantifying how the different policies translate into different inequality levels. One meaningful result is that an exogenous injection in any account (Activities, Factors, Private Institutions) ends in benefiting the richest ones. In our example, the market and production shapes in Italy seem to have a very low power to generate income for the poorest Household groups. Inequality in the personal income distribution seems to be a structural feature of our system. Therefore it can be better assessed with the multiplier approach instead of using only traditional ''synthetic'' measures as Gini, Palma or Zenga indexes.

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1. INTRODUCTION

Inequality in personal income distribution has become recently the focus of wide-ranging attention, by International Institutions, Government Reports and finally, by academic journals across a variety of disciplines. The ways in which higher inequality obstacles growth are now under considerable scrutiny. Social protection and more broadly the Welfare State (e.g. via education and health care) can potentially provide an environment that stimulates rather than undermines economic growth. Inequality in income distribution in industrialized countries is very high and increasing with the rise of per-capita income. A forty-year trend of rising inequality, common to many advanced economies, deserves research to identify the forces which are deeply rooted ''within modern industrial capitalism'' (Solow, 2014). Growing inequality leads to the discussion of issues such as: how different sources of inequality can be analyzed, what are the better measures and what set of concrete proposals can be put forward to reduce inequality.

The ''Lisbon Agenda'' set out by the European Council in Lisbon (march 2000) states that the distribution of national income among individuals may be as important, from the welfare point of view, as the aggregate itself (Keuning and Verbruggen, 2001). This statement implies to find a new analytical framework and new policies guidelines shifting from macro variables to structural ones. The personal income generating process, and its links with inequality, must be deeply analysed. As Atkinson claims, the relationships between ''factor shares... and inequality in the personal distribution of income... is the principal problem of political economy''. It is, then, necessary to ''make a link between incomes at the macroeconomic level (national accounts) and incomes at the level of the household...'' (Atkinson, 2009). This link helps to understand source of inequality in the personal distribution of income. ''In making the link between national income and the income of the household sector, the breakdown by sources is, indeed, necessary since the different sources raise different issues... The link between factor shares and personal distribution is more complex than in the days of classical economists for two important reasons: people have multiple sources of income... and there is considerable inequality within categories of income''. (Atkinson, 2009).

The attempts to link functional to personal income distribution is not an easy task. From a theoretical point of view, the traditional theories on personal income distribution explain the process as the result of many different factors (institutional, socio-economics and demographic). It is very difficult, therefore, to reach a consensus on a shared theory of personal income distribution. A change in the understanding of the inequality sources is needed, also for policy purpose. Policies aimed at reducing inequality ex-ante, and not only ex-post, can be directed to change the composition of the functional distribution of income as depending from labour and capital ownership. Factor shares of national income, the generation and the distribution of income processes should be analysed jointly. Many interesting empirical studies on the inequality trends have been published. Yet despite this endeavour and the growing findings for many countries, we still face very important limitations when we try to analysing and measuring the sources of income inequality. There is still a large gap between national accounts, which focus on macro variables, and inequality datasets, which focus on data coming from household surveys. The totals resulting from micro data usually are not fully consistent with macro ones. Factors explaining capital's income generation are different from those which explain labour's income share.

An important contribution to a better understanding of the links between functional and personal distribution of income comes from studies on the decomposition of inequality in personal income distribution by functional factors. The effects of a rise of the capital income share on personal income distribution depends not only on its amount, but also on the inequality within both capital and labour income distributions and on the correlation between the two. Atkinson (2009) claims that in today's world, where people have earnings from both labour and capital, all effects can be highlighted with a simple decomposition of the squared coefficient of variation of capital and labour incomes and of the correlation between labour and capital income distributions. In a very recent work, Ranaldi (2018) proposes a theoretical framework to examine the relationship between the functional and personal distribution of income introducing the concept of inequality in income composition. ''Inequality in income composition is high when two different sources of income are separately earned by the top and the bottom of the income distribution. On the contrary, it is low when each individual has the same population share of the two source''. The author introduces an indicator to measure income composition inequality, named income-factor concentration index. The framework is then applied to several European countries on basis of EU-SILC data (Ranaldi, 2018).

A very interesting recent work (Zenga and Jedrzejczak, 2020) enlightens the contribution of different income sources to the inequality levels and changes in personal income distribution in two different countries (Italy and Poland). ''The detailed results obtained within the study suggest that inequality decomposition based on the Zenga approach can be helpful for better understanding of the issue of income distribution and income inequality'' (Zenga and Jedrzejczak, 2020). The contribution of the income sources to the total income inequality index has been obtained using a decomposable inequality index, that is the Zenga index (Zenga, Radaelli and Zenga 2012). ''The comparative analysis, which was based on the micro data on household incomes obtained for both countries, helped to reveal several differences in inequality patterns in Poland and Italy, although the countries present similar inequality levels measured by the Gini and Zenga indexes. The differences are especially remarkable when analysing source decomposition: the negligible share of *property income* in average income in Poland (in total and for regions), accompanied by very small shares of *property income* and *pensions* in the overall inequality, helped to recognise income distribution which still can be considered the heritage of a centrally planned economy'' (Zenga and Jedrzejczak, 2020).

An analysis of the process linking functional income distribution to personal one requires not only a decomposition of inequality indexes, but, first of all, a suitable framework for linking factor endowments to household income generation and distribution. The Social Accounting Matrix (SAM), showing the links between the production process and the households' primary income distribution could be, in our opinion, the suitable accounting framework. The SAM is, at the same time, a model aimed at assessing the impact on the structure of production, factorial and personal income distributions, of exogenous changes in macro variables such as exports, certain categories of Government expenditures, and investments. As such, the SAM becomes the basis for a multiplier analysis and for building and calibrating of a variety of applied general equilibrium models. This kind of analysis allows to assess the impact of alternative policies on the income generation, distribution and redistribution process. In addition, the multiplier approach allows to evaluate and to quantify in which ways incomes, equally earned by each group belonging to the household institution, turn into different disposable income levels through the three stages of spending, production and redistribution. We believe that the multiplier matrix thus obtained, can be considered a structural inequality measure. A first version of the methodological framework has been developed in an article published in the 2019 (Civardi and Targetti Lenti, 2019). This work, however, has different applicative purposes, that is highlightening the existence of the links between functional and personal distribution.

The outline of the paper is the following. Section 1 is the Introduction. Section 2 analyses the SAM as an accounting framework. Section 3 discusses the SAM as a simulation model aimed at obtaining the ''accounting multipliers''. Section 4 introduces, as a numerical example, the determination of ''accounting multiplier'' matrix **M** for the Italian economic system. Remarks on the relevance of the proposed framework for a ''structural'' analysis of inequality via the multiplier matrix concludes the paper in Section 5.

2. THE SAM AS AN ACCOUNTING FRAMEWORK

The SAM is a comprehensive, disaggregated, consistent and complete data system that highlights the interdependence that exists within a socioeconomic system (Round, 2003). The SAM captures and shows the entire circular flow of income from its generation to its distribution and its expenditure. It is, basically, a square matrix combining in an accounting framework the flows in value of an economic system, starting from the elementary flows which interrelate the economic units at different level of aggregation. Each transaction or account has its own row and column. The payments (expenditures) are listed in columns and the receipts are recorded in rows. As the sum of all expenditures by a given account (or sub account) must equal the total sum of receipts for the corresponding account, row sums must equal the column sums of the corresponding account.

The circular flow of income from its production to its distribution and its expenditure can be the conceptual starting point for understanding the generating mechanisms of the income distribution process. The primary income distribution is determined by structural relationships linking production activities, factors and households. Hence, if we are interested in understanding how the structure of production influences the income distribution, we can obtain useful insights through a SAM. In the SAM the different flows between factors of production and Institutions must be spelled out, explicitly and quantitatively, at a given level of disaggregation, according to the availability of data.

The SAM can be considered as an extension of the traditional input-output framework. It adds some accounts, not included in the Leontief schema, which allows to take into account explicitly the relationships between factorial distribution of income, primary income distribution to Institutions and final demand. The introduction of accounts referred to Institutions (Households, Private Companies, Government, Rest of the World) allows capturing the primary distribution of income as the link between Factors of production and Institutions which own the different Factors of production.

The secondary distribution of income (disposable income) is also introduced as the result of transfers between different Institutions, mainly between private Institutions and the Government. The disposable income of Institutions is the starting point for sustaining the final demand. In particular, the Household, divided in different socio-economic groups, sustain the demand for consumption. The amount of income, which is not consumed in the current year, is saved and goes into the capital account

3. THE SAM AS A SIMULATION MODEL: THE ACCOUNTING MULTIPLIERS

The SAM is not a model in itself (Round, 2003) but the way of representing the complex relationships between interconnected parts of an economic system. In this sense, a SAM can be used to represent every economic system as a set of accounts and to show all the possible effects, direct, indirect and feed-back, of a change in one particular account on the other accounts. However the inclusion in the SAM of data related to the production side (included data from the labour market) and of data related to the income distribution and to consumption expenditure allows, following Thorbecke (2000), to consider a SAM not only as an accounting framework, but also as a conceptual framework for modelling alternative policies aimed at reducing inequality

If a certain number of conditions are met – in particular, the existence of excess capacity and unemployed or underemployed labour resources – the SAM framework can be used to estimate the impact of exogenous ''injections'' in some accounts on the household income distribution and on the whole system. These injections could be, for instance, an increase in the use of productive factor because of technological change, an increase in the consumption demand for a given commodity, or an increase in Government expenditures following the introduction of new redistributive measures. For any given injection anywhere in the SAM, its influence is transmitted throughout the interdependent SAM system.

A SAM has been frequently used to assess the partial equilibrium effects of shocks on ''real'' variables (production, consumption) of the economy, using it as a multiplier model that treats as endogenous the circular flow of Private Institutions income. This is the main outcome of SAM-based multiplier analysis. The multiplier approach allows assessing and quantifying the different ways by which an income equally earned by each group belonging to the Household institution, turns into different disposable income levels through the four stages of spending, production, distribution and redistribution. Following a Keynesian approach, we can assume that the total level of income of each group determines the level of consumption of different commodities by the Household Institution. The equilibrium solution through the SAM determines the income distribution of the Private Institutions (assumed to be endogenous) consistent with a given production structure under the assumption that the final demand depends on the disposable income of the Endogenous Institutions.

TABLE 1. - *Aggregated SAM with Exogenous and Endogenous Accounts*

	Endogenous Accounts			Exogenous Institutions	Total
	Activities	Factors	Private Institutions		
Activities	$\mathbf{S_{11}}$	**0**	$\mathbf{S_{13}}$	$\mathbf{x_1}$	$\mathbf{t_1}$
Factors	$\mathbf{S_{21}}$	**0**	**0**	$\mathbf{x_2}$	$\mathbf{t_2}$
Private Institutions	**0**	$\mathbf{S_{32}}$	$\mathbf{S_{33}}$	$\mathbf{x_3}$	$\mathbf{t_3}$
Exogenous Institutions	$\mathbf{l'_1}$	$\mathbf{l'_2}$	$\mathbf{l'_3}$	$\mathbf{x_4}$	$\mathbf{t_4}$
Total	$\mathbf{t'_1}$	$\mathbf{t'_2}$	$\mathbf{t'_3}$	$\mathbf{t'_4}$	

Different components can be identified in Table 1. Some important accounting relations can now be derived from the structure of the SAM. $\mathbf{S_{ij}}$ are submatrices representing transactions occurring between endogenous institutions. In particular, the matrix $\mathbf{S_{11}}$ $(n \times n)$ represents the inter-industry transactions and measures the intermediate consumption by the n endogenous production activities. Matrix $\mathbf{S_{13}}$ $(n \times h)$ indicates final consumption by the h private institutions. The matrix $\mathbf{S_{21}}$ $(m \times n)$ provides the distribution of value added from the n productive sectors to the m endogenous production factors. Matrix $\mathbf{S_{32}}$ $(h \times m)$ describes the process of distribution of income from the m factors to the h endogenous private institutions. Finally, the matrix $\mathbf{S_{33}}$ $(h \times h)$ gives values of transfers occurring among endogenous private institutions. The zero elements of the submatrices of endogenous accounts indicate that there are no transactions among these accounts or that information about monetary transactions between the two accounts are not available.

Column vector $\mathbf{x=[x_1\ x_2\ x_3]}$ $(n+m+h) \times 1)$ represents the exogenous components of the system from which exogenous injections are generated. $\mathbf{x_1}$ is the exogenous expenditure in national goods from the Government and from the RoW (Rest of the World); $\mathbf{x_2}$ is the amount of the value added transferred from the RoW and $\mathbf{x_3}$ is the value of transfers from the Government and the RoW to the private institutions.

Row vector $\mathbf{l=[l'_1\ l'_2\ l'_3]}$ $(1 \times (n+m+h))$ represents the *leakages*, i.e. the flows of income from endogenous institutions to exogenous ones. $\mathbf{l'_1}$ $(1 \times n)$ is the flow of income from activities to the RoW for imports; $\mathbf{l'_2}$ $(1 \times m)$ is the amount of value added transferred to the RoW; $\mathbf{l'_3}$ $(1 \times h)$ are the transfers of income from private institutions to the Government (indirect income taxes) and to the RoW. Finally, column vector $\mathbf{x_4}$ (1×1) provides all the possible transactions among exogenous accounts.

With reference to the SAM of Table 1, equations expressing the process of generation of the total value added can be written out in explicit form following a Keynesian approach. The equation (1) indicates, first of all, that the value of total production of the n activities ($\mathbf{t_1}$) must equal the final total demand for intermediate commodities ($\mathbf{S_{11}}$), the demand for consumption from private institutions ($\mathbf{S_{13}}$) and the residual component of the final demand $\mathbf{x_1}$. Equation (2) indicates that total factorial income ($\mathbf{t_2}$) should be equal to value added produced by the endogenous activities and then distributed to factors ($\mathbf{S_{21}}$) plus the exogenous component $\mathbf{x_2}$. Equation (3) indicates that the total disposable income ($\mathbf{t_3}$) resulting from the primary and secondary distribution process is equal to the income obtained from the factors ($\mathbf{S_{32}}$) and from the redistribution process within the endogenous institutions ($\mathbf{S_{33}}$) plus the income from the exogenous institutions $\mathbf{x_3}$.

$$\mathbf{t_1} = \mathbf{S_{11}}\,\mathbf{e_1} + \mathbf{S_{13}}\,\mathbf{e_3} + \mathbf{x_1} \qquad (1)$$
$$\mathbf{t_2} = \mathbf{S_{21}}\,\mathbf{e_1} + \mathbf{x_2} \qquad (2)$$
$$\mathbf{t_3} = \mathbf{S_{32}}\,\mathbf{e_2} + \mathbf{S_{33}}\,\mathbf{e_3} + \mathbf{x_3} \qquad (3)$$

$\mathbf{e_1}$ $(n \times 1)$, $\mathbf{e_2}$ $(m \times 1)$ $\mathbf{e_3}$ $(h \times 1)$ are unit vectors

The matrices of expenditure coefficients $\mathbf{A_{jk}}$ are obtained dividing the matrices $\mathbf{S_{jk}}$ by the diagonal matrix $\mathbf{\hat{t}_k}$ whose elements are the components of $\mathbf{t'_k}$.

$$\mathbf{A_{jk}} = \mathbf{S_{jk}}\ (\mathbf{\hat{t}_k})^{-1} \qquad (4)$$

The hypothesis of fixed expenditure coefficients resulting from $\mathbf{A_{jk}}$ is consistent with the assumptions of the linear expenditure system developed by Stone (1954) for which there is widespread empirical support. The normalisation of the transactions matrices $\mathbf{S_{jk}}$ allows the constraints relating to row and column totals of the SAM in Table 1 to be rewritten isolating the group of the r (three in our case) endogenous accounts from the exogenous ones. We can, thus, write

$$\mathbf{t_{end.}} = \mathbf{A}\ \mathbf{t_{end.}} + \mathbf{x_{end.}} \qquad (5)$$
$$\mathbf{t_4} = \mathbf{ll'_1 t_1} + \mathbf{ll'_2 t_2} + \mathbf{ll'_3 t_3} + \mathbf{x_4} \qquad (6)$$

where $\mathbf{ll'_k} = \mathbf{l'_k}\left(\mathbf{\hat{t}_k}\right)^{-1}$

Equation (6) indicates that the equilibrium values of the accounts relating to the exogenous Institutions is achieved once the endogenous accounts are in equilibrium. Finally, considering the previous equations and the accounting principle that total receipts must equal total outlays, it follows that, in aggregate, total injections into the system must equal total leakages (Pyatt and Round, 1979).

The conditions express in (6) allow that only (5) is taken into consideration and it can be rewritten as

$$\mathbf{t_{end.}} = (\mathbf{I} - \mathbf{A})^{-1} \mathbf{x_{end.}} = \mathbf{M}\mathbf{x_{end.}} \quad (7)$$
$$\mathbf{M} = (\mathbf{I} - \mathbf{A})^{-1} \quad (8)$$

The equation (7) indicates that the vector $\mathbf{t_{end.}}$ of the income of each endogenous account (i.e production activity incomes $\mathbf{t_1}$, factor incomes $\mathbf{t_2}$, and institution incomes $\mathbf{t_3}$, as shown in Table 1) can be obtained multiplying the vector of exogenous injections $\mathbf{x_{end.}}$ by the generalised inverse of matrix **A**, that is $(\mathbf{I}\text{-}\mathbf{A})^{-1}$. The matrix **M** in (7) shows the overall effects on each element of the endogenous accounts resulting from the direct and indirect transfer processes generated by an initial increase in each of the three exogenous components $\mathbf{x_1}$, $\mathbf{x_2}$, $\mathbf{x_3}$. The matrix has been introduced by Pyatt and Round (1979) in a seminal contribution and can be interpreted as a simplified model of the actual way the system is working. The total, direct and indirect, effects of the injection $\mathbf{x_{end}}$ on the endogenous accounts, i.e. the total outputs of the different production activities and the incomes of various factors and private institution are, then, obtained as the result of a multiplier process.

In order to capture how the matrix of the global multiplier $\mathbf{M} = (\mathbf{I}\text{-}\mathbf{A})^{-1}$ works to generate a new distribution of income to the endogenous institutions as a response to an exogenous injection it is useful to write out in explicit form equation (5). Following Thorbecke (2000) and considering the structure of the aggregate SAM in Table 1, we can write:

$$\mathbf{t_1} = \mathbf{A_{11}}\,\mathbf{t_1} \qquad\qquad + \mathbf{A_{13}}\,\mathbf{t_3} + \mathbf{x_1} \quad (9)$$
$$\mathbf{t_2} = \mathbf{A_{21}}\,\mathbf{t_1} + \qquad\qquad\qquad + \mathbf{x_2} \quad (10)$$
$$\mathbf{t_3} = \qquad\qquad \mathbf{A_{32}}\,\mathbf{t_2} + \mathbf{A_{33}}\,\mathbf{t_3} + \mathbf{x_3} \quad (11)$$

The equations from (9) to (11) can be rewritten as:

$$\mathbf{t_1} = (\mathbf{I}\text{-}\mathbf{A_{11}})^{-1}\mathbf{x_1} + (\mathbf{I}\text{-}\mathbf{A_{11}})^{-1}\mathbf{A_{13}}\,\mathbf{t_3} \quad (12)$$
$$\mathbf{t_2} = \mathbf{A_{21}}\,\mathbf{t_1} + \mathbf{x_2} \quad (13)$$
$$\mathbf{t_3} = (\mathbf{I}\text{-}\mathbf{A_{33}})^{-1}\mathbf{x_3} + (\mathbf{I}\text{-}\mathbf{A_{33}})^{-1}\mathbf{A_{32}}\,\mathbf{t_2} \quad (14)$$

The set of equations from (12) to (14) can be represented graphically in Figure 1. Remembering that:

– $\mathbf{x_1}$ is the exogenous final demand from Government consumptions, exports and investments,
– $\mathbf{x_2}$ is the exogenous final demand for factors from Government consumption, export and investment demand,
– $\mathbf{x_3}$ is the exogenous transfers from Government and remittances from abroad toward the Institutions,

the loop of the Figure 1 (Torbecke, 2000) shows clearly and explicitly the mechanisms through which the multiplier process operates as the result of different exogenous injections.

The starting point of the full loop is an exogenous increase (injection) of export, Government, or exogenous demand of investment $\mathbf{x_1}$. This generates a rise in the output of the corresponding production activity equal to $(\mathbf{I}-\mathbf{A_{11}})^{-1}\mathbf{x_1}$. In turn, the employment of additional factors of production, as input of an increase of output, generate a stream of value added $\mathbf{A_{21}t_1}$. This value becomes income from factors in addition to any exogenous income earned by factors coming from other regions or from abroad and from the Government, namely $\mathbf{x_2}$.

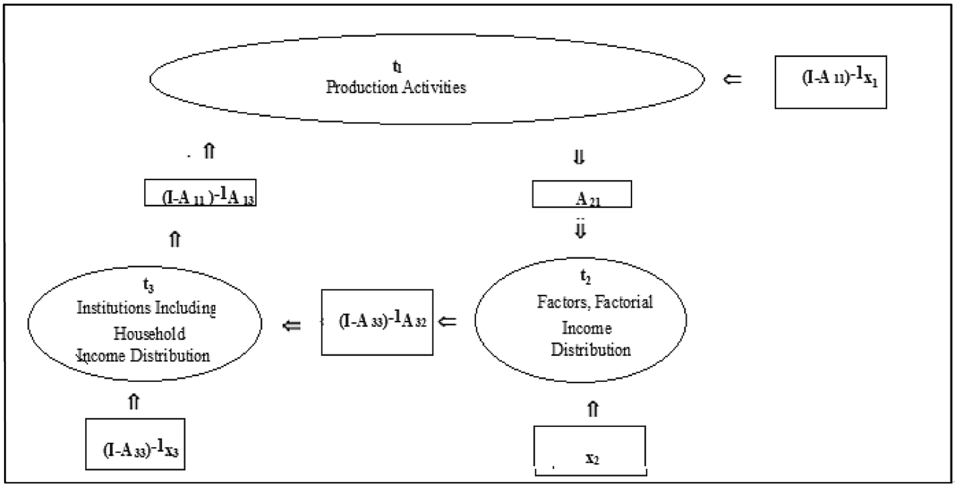

FIGURE 1. - *Multiplier Process among endogenous account*

In the next link, households and companies receive primary income ($\mathbf{A_{32}}$) and transfers within the household sector ($\mathbf{A_{33}}$) as well as exogenous Government subsidies and transfer payments and remittances from other regions and abroad, i.e. $(\mathbf{I}-\mathbf{A_{33}})^{-1}\mathbf{x_3}$.

Finally, the triangle is closed through the pattern of households and companies expenditures on commodities which translate into new production and additional flow of income accruing to the production activities equal to $\mathbf{t_1} = (\mathbf{I}-\mathbf{A_{11}})^{-1}\ \mathbf{A_{13}}$.

The loop in Figure 1 can be considered a generalization of the Leontief model. The added component, in comparison to the I-O model, is the effect of the vector $\mathbf{t_3}$, which represents the personal income distribution, on the consumption of the various socioeconomic groups through the matrix $\mathbf{A_{13}}$. This matrix expresses the consumption pattern of each group of households. In the traditional Leontief model the households' consumption vector is included in the final demand as an exogenous component.

The accounting multipliers obtained using the SAM as a linear model allow to capture the structural features of income distribution and the interrelations operating between the households' groups and with the other Institutions and Activity Sectors.

Because in this work we focus on the distribution of households income, we will separate and distinguish Private Institutions between Households and Companies.

The elements of the matrix $\mathbf{M}$ (Table 2) related to Households Institution (sub matrices $\mathbf{M_{31}}$, $\mathbf{M_{32}}$, $\mathbf{M_{33}}$ and $\mathbf{M_{34}}$) have the meaning, at a disaggregated level, of a Keynesian multiplier. Its values depend on the linkages built in the SAM (consumption expenditure, input-output relationships, value added distributed to different households groups according to their ownership of the production factors).

TABLE 2. - *Multiplier Matrix M*

		Endogenous Accounts				Total
		Activities	Factors	Private Institutions		
				Households	Companies	
Activities		$\mathbf{M_{11}}$	$\mathbf{M_{12}}$	$\mathbf{M_{13}}$	$\mathbf{M_{14}}$	$\mathbf{t_1}$
Factors		$\mathbf{M_{21}}$	$\mathbf{M_{22}}$	$\mathbf{M_{23}}$	$\mathbf{M_{24}}$	$\mathbf{t_2}$
Private Institutions	Households	$\mathbf{M_{31}}$	$\mathbf{M_{32}}$	$\mathbf{M_{33}}$	$\mathbf{M_{34}}$	$\mathbf{t_3}$
	Companies	$\mathbf{M_{41}}$	$\mathbf{M_{42}}$	$\mathbf{M_{43}}$	$\mathbf{M_{44}}$	$\mathbf{t_4}$
Total		$\mathbf{t'_1}$	$\mathbf{t'_2}$	$\mathbf{t'_3}$	$\mathbf{t'_4}$	

Focusing on the determination of the income distributed within the Households, the corresponding $\mathbf{t_3}$ vector is given by:

$$\mathbf{t_3 = M_{31}\, x_1 + M_{32}\, x_2 + M_{33}\, x_3 + M_{34}\, x_4} \tag{15}$$

Equation (15) allows us to determinate the total income of each Households group by the $\mathbf{M_{31}}$ $\mathbf{M_{32}}$ $\mathbf{M_{33}}$ and $\mathbf{M_{34}}$ multipliers (Bottiroli Civardi, 1990).

The sum of the elements of the matrix $\mathbf{M_{31}}$ indicates the increase in the overall income of Households as a whole due to an exogenous injection of one unit in the income of each Activities account. The corresponding sums concerning $\mathbf{M_{32}}$ $\mathbf{M_{33}}$ and $\mathbf{M_{34}}$ matrices indicate the increase in the overall income of Households as a whole due to an exogenous injection of one unit in the income of each Factors or of each Private Institutions.

The column totals of each $\mathbf{M_{3j}}$ are real income multipliers. They indicate by how much the income of Households as a whole would rise if the income of the corresponding endogenous accounts (Activities, Factor, Private Institutions) would exogenously increase by one unit. The row totals indicates the multiplier effect on the income of each Households group if the income of each Activity, Factor or Private Institution would increase by one unit.

4. A NUMERICAL EXAMPLE: THE DETERMINATION OF THE "ACCOUNTING MULTIPLIER" **M** FOR THE ITALIAN ECONOMIC SYSTEM

The meaning and the relevance of the multiplier approach in using the SAM as a simulation model can be illustrated with a numerical example, referred to the Italian economy. The aim is to obtain the multiplier matrix **M** starting from the 1984 Sam for Italy determined by Bottiroli Civardi, Chiappero and Targetti Lenti (1990) in a past research work. Then we must introduce the submatrices we will use in the process of building the matrix **M**. The construction of the 1984 Italian SAM required an extensive processing of data drawn from different sources and the introduction of many simplifying hypothesis. As mentioned in Section 3, in order to highlight the potentiality of the SAM as a simulation model, some of the accounts of the base SAM have been aggregated in endogenous and exogenous ones. Endogenous Institutions are Households and Companies. All the other accounts of the SAM were aggregated into the vector of Exogenous Institutions (Table 1).

The transactions values in the matrix $\mathbf{S}_{32}$ of the SAM (Table 3), at the crossing between factors and household accounts are ''gross'' or market incomes. The data which allowed estimating these flows, were drawn by the annual Survey of the Bank of Italy (Banca d'Italia, 1985). The survey, however, collects only the values of disposable income (net income), i.e. the income that includes the impact of existing tax and transfer systems. Therefore it has been necessary, to estimate the gross income in some subsequent steps, with a micro-simulation model developed to simulate taxes, benefits, social insurance contributions and other transfers received. At the end of the process, the net incomes have been turned into gross income, taking into account the features of the Italian fiscal system in the 1984. For details of this quite complex methodology see: Bottiroli Civardi *et al.* (1990). Households have been classified in 10 groups (deciles of population) according to their level of disposable (net) income).

TABLE 3. - *Matrix $\boldsymbol{S}_{32}$ (SAM Italy 1984) Transaction values in millions of lira**

	Employees income	Self-empl. income	Product capital	Housing capital	Financial capital	Total
1^ decile	5286	883	390	1224	483	8266
2^ decile	14089	2706	1481	1644	766	20686
3^ decile	18936	3433	2284	2060	1045	27758
4^ decile	23365	4121	2958	2156	1269	33869
5^ decile	30126	3781	2647	2348	1695	40597
6^ decile	32619	5102	3500	2904	1712	45837
7^ decile	39218	5583	3531	2802	2366	53500
8^ decile	46706	7187	6027	3638	2875	66433
9^ decile	53719	11620	8836	4434	4664	83273
10^ decile	75595	23748	26491	7688	9776	143298
Companies	0	0	60096	4254	0	64350
Total	339659	68164	118241	35152	26651	587867

*The matrix is bordered with the row and column of totals

TABLE 4. - *Matrix S_{32} column percentages of decile' incomes from Factors**

	Employees income	Self-empl. income	Product capital	Housing capital	Financial capital	Total
1^ decile	1.56	1.30	0.33	3.48	1.81	1.41
2^ decile	4.15	3.97	1.25	4.68	2.87	3.52
3^ decile	5.58	5.04	1.93	5.86	3.92	4.72
4^ decile	6.88	6.05	2.50	6.13	4.76	5.76
5^ decile	8.87	5.55	2.24	6.68	6.36	6.91
6^ decile	9.60	7.48	2.96	8.26	6.42	7.80
7^ decile	11.55	8.19	2.99	7.97	8.88	9.10
8^ decile	13.75	10.54	5.10	10.35	10.79	11.30
9^ decile	15.82	17.05	7.47	12.61	17.50	14.17
10^ decile	22.26	34.84	22.40	21.87	36.68	24.38
Companies	0.00	0.00	50.83	12.10	0.00	10.95
Total	100.00	100.00	100.00	100.00	100.00	100.00

*The matrix is bordered with the row and column of totals

The matrix $\mathbf{S_{32}}$ in Table 4 shows the column percentages of decile' incomes from Factors. This values are a signal of the degree of inequality of income from decile' factors ownership. On average, the income of the last decile weights almost a quarter of the total while the first two deciles together get only the 4.9%. This "great divide" is common more or less to each Factor.

TABLE 5. - *Matrix S_{32}: row percentages of decile' incomes from Factors**

Companies	Employees income	Self-empl. income	Product capital	Housing capital	Financial capital	Total
1^ decile	63.95	10.68	4.72	14.81	5.84	100.00
2^ decile	68.11	13.08	7.16	7.95	3.70	100.00
3^ decile	68.22	12.37	8.23	7.42	3.76	100.00
4^ decile	68.99	12.17	8.73	6.37	3.75	100.00
5^ decile	74.21	9.31	6.52	5.78	4.18	100.00
6^ decile	71.16	11.13	7.64	6.34	3.73	100.00
7^ decile	73.30	10.44	6.60	5.24	4.42	100.00
8^ decile	70.31	10.82	9.07	5.48	4.33	100.00
9^ decile	64.51	13.95	10.61	5.32	5.60	100.00
10^ decile	52.75	16.57	18.49	5.37	6.82	100.00
Companies	0.00	0.00	93.39	6.61	0.00	100.00
Total	57.78	11.60	20.11	5.98	4.53	100.00

*The matrix is bordered with the row and column of totals

TABLE 6. - *Matrix $\boldsymbol{S_{33}}$ (SAM Italy 1984) Transaction values in millions of lira* *

	1^ decile	2^ decile	3^ decile	4^ decile	5^ decile	6^ decile	7^ decile	8^ decile	9^ decile	10^ decile	Total
1^ decile	1.0116	0.0122	0.0122	0.0116	0.0104	0.0112	0.0111	0.0107	0.0101	0.0082	1.1091
2^ decile	0.0281	1.0296	0.0297	0.0283	0.0254	0.0273	0.0272	0.0260	0.0245	0.0198	1.2660
3^ decile	0.0376	0.0396	1.0397	0.0380	0.0341	0.0366	0.0364	0.0347	0.0327	0.0265	1.3561
4^ decile	0.0455	0.0481	0.0482	1.0461	0.0413	0.0444	0.0442	0.0421	0.0397	0.0322	1.4318
5^ decile	0.0530	0.0560	0.0562	0.0537	1.0483	0.0519	0.0516	0.0492	0.0464	0.0376	1.5038
6^ decile	0.0610	0.0644	0.0646	0.0617	0.0554	1.0596	0.0593	0.0565	0.0533	0.0432	1.5790
7^ decile	0.0701	0.0740	0.0743	0.0710	0.0638	0.0686	1.0682	0.0649	0.0612	0.0496	1.6658
8^ decile	0.0883	0.0933	0.0936	0.0895	0.0804	0.0864	0.0859	1.0818	0.0772	0.0625	1.8390
9^ decile	0.1132	0.1195	0.1199	0.1146	0.1029	0.1105	0.1099	0.1047	1.0987	0.0800	2.0738
10^ decile	0.2042	0.2155	0.2161	0.2066	0.1854	0.1993	0.1980	0.1887	0.1778	1.1441	2.9357
Total	1.7127	1.7522	1.7545	1.7212	1.6474	1.6957	1.6920	1.6593	1.6215	1.5037	16.7601

*The matrix is bordered with the row and column of totals

From Table 5 we can see that share of employees income in the 5th up to the 8th decile, is higher than 70%. In the first four deciles, instead, is lower, and it is still lower in the last two deciles, accounting for only 64.5% and 52.8% respectively. Table 5 shows also that the income shares from self-employment and from productive capital assume the highest values in the last decile, even if for self-employment the range of values are smaller (from 9.3% to 16.6%) then for productive capital (from 4.7% to 18.5%).

The matrix $\mathbf{S_{33}}$ (Table 6) describes the transfers occurring among endogenous private institutions. Because the information about monetary transactions between deciles were not available, the values of the cells are zero. Only the values of the row and the column referred to the Companies are included.

In Table 7, $\mathbf{x_1}$ is the exogenous vector of expenditure in national goods from the Government and the Row, the vector $\mathbf{x_2}$ is the amount of the value added transferred from the RoW to Factors. The vector $\mathbf{x_3}$ is the amount of the transfers to the Households from the Government and of the remittances from abroad. The vector $\mathbf{t_3}$ indicates the total disposable income of each decile.

By (15), it is possible to determinate the vector $\mathbf{t_3}$ of total disposable income of each Households group by the $\mathbf{M_{31}}$ $\mathbf{M_{32}}$ $\mathbf{M_{33}}$ and $\mathbf{M_{34}}$ multipliers (Table 2). In this numerical example, we focus only on the increase in the overall income of Households due to an exogenous injection in the income of one (or more) Factors or Private Institutions, so we introduce and discuss only the matrices $\mathbf{M_{32}}$ $\mathbf{M_{33}}$.

Table 8 shows the values of $\mathbf{M_{32}}$ accounting multipliers for a unit exogenous injection $\mathbf{x_2}$, that is for an unit exogenous injection of income toward each different factors.

This matrix reflects the links between functional and personal income distribution, by quantifying how any injection/change in the factors income composition ends up in a differentiated change of the household income distribution. It highlights, then, also the structural and endogenous level of inequality. In a general equilibrium setting, related to the SAM use, this inequality can be considered as that equilibrium level compatible with the existing structure of the productive activities, the composition of Household consumption expenditure, and the links that interwoven the Private Institutions. These structural features depend on the demographic composition of the Households, on their ownership of the production factors and on the technological features of the production process.

A reading by row of the values in Table 8 shows the effects on each decile's disposable income due to an exogenous increase of one unit directed toward the Factor in column. All the values in the column of rows Total show a monotonically upward trend. The value for the 1st decile (0.1273) is rather small and it indicates the reduced power of injections to the factors to generate income for the poorest, while the multiplier effect in favour of the last decile appears to be stronger (17 times the first).

Column totals shows the effects produced on the Households as a whole by one unit injection to the column's Factor. Employees labor does not seem to play a domi-

TABLE 7. - *(SAM Italy 1984) Transaction values in millions of lira**

Vector $\mathbf{x_1}$

Agriculture	4303
Industry	235298
Trade	12291
Transports	12889
Credit and Insurance	949
Public Administration	136933
Other Services	4548
Total	407211

Vector $\mathbf{x_2}$

Employees income	-571
Self-empl. income	0
Product capital	-16373
Housing capital	0
Financial capital	0
Total	-16944

Vector $\mathbf{x_3}$

1^ decile	15289
2^ decile	13831
3^ decile	15645
4^ decile	14175
5^ decile	14155
6^ decile	14155
7^ decile	14176
8^ decile	13844
9^ decile	14224
10^ decile	21405
Companies	35583
Total	186482

Vector $\mathbf{t_3}$

1^ decile	23770
2^ decile	35019
3^ decile	44071
4^ decile	48863
5^ decile	55858
6^ decile	61173
7^ decile	69172
8^ decile	82047
9^ decile	99626
10^ decile	167904
Companies	117586
Total	805089

nant multiplying effect as compared to Self-employment and Financial capital: 1.6391 compared to 1.6163 and to 1.6104. However, its value is the highest one.

TABLE 8. - *Matrix M_{32} (SAM Italy 1984)**

	Employees income	Self-empl. income	Product capital	Housing capital	Financial capital	Total
1^ decile	0.0259	0.0229	0.0095	0.0410	0.0280	0.1273
2^ decile	0.0666	0.0639	0.0276	0.0645	0.0528	0.2755
3^ decile	0.0894	0.0828	0.0395	0.0827	0.0714	0.3657
4^ decile	0.1096	0.0998	0.0495	0.0912	0.0866	0.4368
5^ decile	0.1363	0.1014	0.0516	0.1022	0.1091	0.5007
6^ decile	0.1508	0.1276	0.0628	0.1228	0.1165	0.5805
7^ decile	0.1784	0.1426	0.0686	0.1272	0.1489	0.6658
8^ decile	0.2168	0.1819	0.0993	0.1629	0.1837	0.8447
9^ decile	0.2597	0.2684	0.1360	0.2025	0.2720	1.1385
10^ decile	0.4055	0.5248	0.3316	0.3564	0.5415	2.1598
Total	1.6391	1.6163	0.8761	1.3534	1.6104	7.0953

*The matrix is bordered with the row and column of totals

The $\mathbf{M_{33}}$ (Table 9) multiplier matrix allows to quantify the different ways by which an income equally earned by each decile turns into different levels of income through the steps of spending and producing in a market economy, that is through

TABLE 9. - *Matrix* $\mathbf{M}_{33}$ *(SAM Italy 1984)**

	1^ decile	2^ decile	3^ decile	4^ decile	5^ decile	6^ decile	7^ decile	8^ decile	9^ decile	10^ decile	Total
1^ decile	1.0116	0.0122	0.0122	0.0116	0.0104	0.0112	0.0111	0.0107	0.0101	0.0082	1.1091
2^ decile	0.0281	1.0296	0.0297	0.0283	0.0254	0.0273	0.0272	0.0260	0.0245	0.0198	1.2660
3^ decile	0.0376	0.0396	1.0397	0.0380	0.0341	0.0366	0.0364	0.0347	0.0327	0.0265	1.3561
4^ decile	0.0455	0.0481	0.0482	1.0461	0.0413	0.0444	0.0442	0.0421	0.0397	0.0322	1.4318
5^ decile	0.0530	0.0560	0.0562	0.0537	1.0483	0.0519	0.0516	0.0492	0.0464	0.0376	1.5038
6^ decile	0.0610	0.0644	0.0646	0.0617	0.0554	1.0596	0.0593	0.0565	0.0533	0.0432	1.5790
7^ decile	0.0701	0.0740	0.0743	0.0710	0.0638	0.0686	1.0682	0.0649	0.0612	0.0496	1.6658
8^ decile	0.0883	0.0933	0.0936	0.0895	0.0804	0.0864	0.0859	1.0818	0.0772	0.0625	1.8390
9^ decile	0.1132	0.1195	0.1199	0.1146	0.1029	0.1105	0.1099	0.1047	1.0987	0.0800	2.0738
10^ decile	0.2042	0.2155	0.2161	0.2066	0.1854	0.1993	0.1980	0.1887	0.1778	1.1441	2.9357
Total	1.7127	1.7522	1.7545	1.7212	1.6474	1.6957	1.6920	1.6593	1.6215	1.5037	16.7601

* The matrix is bordered with the row and column of totals

the entire loop process (Figure 1). It highlights, then, how an exogenous transfer, for instance by the Government, can turn out at the end of the process in differentiated incomes for each decile according to the multipliers' values. The row totals of $\mathbf{M_{33}}$ express the inequality in the Households' income distribution that can be considered ''structural''. All these values show a monotonically upward trend. The value for the 1st decile (1.1091) is the smaller and it indicates the reduced potential of the system to convert an exogenous injection in income for the poorest. On the opposite, the multiplier effect in favour of the last decile (2.9357) seems to be particularly strong.

The column totals indicate the income generating power of each decile toward the households as a whole. The multipliers values show that the first four deciles, and particularly the second and the third, have the highest multiplying power. This multiplying capacity could end up to have a regressive effect.

Diagonal elements represent the income multiplier within each deciles generated by an additional unit of income exogenously attributed to the group itself. They are obviously all higher than one and show a monotonically growing trend from the first to the last decile. This means that, for an equal exogenous injection of additional income, the final effect within the poorest deciles is always weaker than within the richest ones. The poorest deciles, furthermore, have a lower power to generate income for themselves than for generating income for the households as a whole.

5. THE IMPACT OF SOME SIMULATED REDISTRIBUTIVE POLICIES ON HOUSEHOLD INCOME DISTRIBUTION

The power of the multiplicative approach can be better assessed by using it as a tool to measure the impact of different redistributive policies. We tested the following seven alternative hypothesis of exogenous injection toward different deciles or different factors. For comparing their impacts, it is necessary that the total amount transferred by Government is the same for all policies. In particular, we have assumed that the total injection amount is 2 billion of lira.

The hypotheses tested are:

Hypothesis 1: An exogenous injection equal to 2000 units (2 billion of lira) to the first decile;

Hypothesis 2: An exogenous injection equal to 2000 units to the first two deciles (1 billion of lira to each one);

Hypothesis 3: An exogenous injection equal to 2000 units to the first four deciles (500 million of lira to each one);

Hypothesis 4: An exogenous injection equal to 2000 units (2 billion of lira) to the tenth decile;

Hypothesis 5: An exogenous injection equal to 3000 units (3 billion of lira) to the first decile and together a negative injection of -1000 units (-1 billion of lira) to the tenth decile;

Hypothesis 6: An exogenous injection equal to 2000 units (2 billion of lira) to the ''Employees income labour'' factor;

Hypothesis 7: An exogenous injection equal to 2000 units (2 billion of lira) to the two factors ''Employees income labour'' and ''Self-employment income'' (1 billion of lira to each).

The ex-post inequality in the distribution of income between groups can be considered only as the outcome of the mechanism explicitly considered in the model. Therefore, also the ''occurring'' deciles' income increases must be considered as a signal of the direction of changes rather than as actual values. Table 10 shows the distributions of the final households' income distribution (at the end of the loop of Figure 1) for each of the tested hypotheses. The column labelled Hyp.1 highlights the distribution of income of each decile following a transfer by the Government of 2 billion of lira to the first decile (equal to 8.41% of its ''starting'' income). The values show not only a gain of 8.51% (+0.1 percentage points further gain compared to the initial injection) for the first decile but also an increase (+ 1.48%) in the income of the first four deciles, a gain of 0.23% for the other deciles (the last 60%) and of 0.25% for the top 10%. The columns related to Hyp.2 and Hyp.3 indicate for the poorest decile a gain of 4.1% (+0.1 percentage points of further gain compared to the initial injection of 1 billion lira) and of 2.20% (+0.1 percentage points of further gain compared to the initial injection of 1/2 billion of lira). The first four deciles gain in both hypotheses 1.48% (+0.17 percentage points of further gain compared to the injection). The last 60% gains in both hypotheses 0.23% and the richest decile gains 0.25%.

TABLE 10. - *Vectors t_3 of total disposable income resulting from the simulations (millions of lira)*

	Base SAM	Hyp. 1	Hyp. 2	Hyp. 3	Hyp. 4	Hyp. 5	Hyp. 6	Hyp. 7
1^ dec.	23770	25793	24794	24294	23786	26797	23822	23819
2^ dec.	35019	35075	36077	35577	35059	35083	35152	35150
3^ dec.	44071	44146	44148	44648	44124	44157	44250	44243
4^ dec.	48863	48954	48957	49457	48927	48967	49082	49072
5^ dec.	55858	55964	55967	55967	55933	55980	56131	56096
6^ dec.	61173	61295	61298	61299	61259	61313	61475	61451
7^ dec.	69172	69312	69316	69317	69271	69333	69529	69493
8^ dec.	82047	82224	82229	82229	82172	82250	82481	82446
9^ dec.	99626	99852	99859	99860	99786	99885	100145	100154
10^ dec.	167904	168312	168324	168325	170192	167373	168715	168834
Total	687503	690928	690968	690973	690510	691137	690781	690758

The different gains of each decile, in the three alternatives hypotheses (that could correspond to some income supporting policies), end up in a decrease in the inequality indices as shown in Table 11. The Palma index (2006) presents the highest decrease (-1.22 percentage points). This index, however, because of its calculation method, gives the same result for all the three hypotheses. Both Gini (1914) and Zenga (2007) indices, instead, present the highest reduction in the hypothesis 1 and the minimum one in the hypothesis 3. In this case the decrease in the Gini index is higher than in the Zenga one (-0.81 percentage points compared to the ''starting'' distribution for the Gini index and -0.57 for Zenga index).

The fourth hypothesis, consisting in a regressive policy, ends up with a 1.36% increase of the top decile income and in slights increases for both the first decile and the first four deciles (+0.07% and +0.11%). The impact on the income of the households as a whole, in this case, is the lowest: only 0.44% compared to the 0.50% of the first three hypotheses. As a result, the level of inequality increases as indicated by the three indices in Table 11. This means that the increase of income of the poorest decile resulting from the multiplier effects of circular flow of income is not enough to compensate the effects on inequality of the transfers direct only towards the last decile. In this case the Palma index is the most ''reactive'' (+1.25 percentage points) while that of Zenga shows the lowest variation (+0.39 percentage points).

TABLE 11 - *Inequality indices*

	Inequality Indices					
	Values			Rates of change %		
	Gini	Zenga	Palma	Gini	Zenga	Palma
Base Sam	0.3241	0.6048	1.1066	0.00	0.00	0.00
Hypothesis 1	0.3205	0.5983	1.0932	-1.11	-1.08	-1.22
Hypothesis 2	0.3208	0.5997	1.0932	-1.01	-0.84	-1.22
Hypothesis 3	0.3215	0.6013	1.0932	-0.81	-0.57	-1.22
Hypothesis 4	0.3262	0.6071	1.1204	0.65	0.39	1.25
Hypothesis 5	0.3176	0.5938	1.0798	-2.00	-1.81	-2.43
Hypothesis 6	0.3244	0.6052	1.1077	0.09	0.06	0.10
Hypothesis 7	0.3246	0.6053	1.1087	0.15	0.09	0.18

With reference to the Hypothesis 5, which consists in an exogenous reduction (tax collection) equal to 0.60% of its income for the latest decile and, simultaneously, in an increase of 12.62% in the income of the first decile, Table 10 indicates a gain of 2.16% for the first four deciles and a loss of 0.32% for the last one.

However, the loss for the last decile is 0.28 percentage points lower than the planned income withdrawal. Mainly due to the greater multiplier power of the poorest decile, the effect on income of total households is the most positive (+ 0.53%). Here the reduction in inequality is the highest (-2.43% in the Palma index, -2.00% in the Gini index and -1.81% in Zenga index).

The Hypotheses 6 and 7 allow assessing the effects, on the Households income distribution and on its inequality, of an exogenous transfer given to the production factors. The Hypothesis 6 (injection of 2 billion lira to the employees income labour factor) could correspond to a policy of reducing the tax wedge. The Hypothesis 7 (injection of 1 billion lira both to the employees income labour and to the Self-employment income) could correspond to policies for reducing taxes on the labour factor. On the income of the households as a whole both policies have a positive effect not much lower than that of the previous hypotheses (respectively +0.48% and +0.47%), in any case higher than that of Hypothesis 4 (of the regressive type). According to Hypothesis 6, the income of the first decile would increase by 0.22% and that of the last by 0.48%. With Hypothesis 7, these increases would be respectively 0.21% and 0.55%. In both cases the inequality increases even if only slightly (respectively +0.09% according to the Gini index with hypothesis 6 and +0.15% with hypothesis 7). The Palma index is the most ''reactive'' to the changes caused by these exogenous injections. Policies aimed at increasing the income of the employees' labour factor seem to benefit above all the middle deciles which, as shown in Table 5, are those with the highest shares of this factor. Furthermore, from Table 4, it can be seen that the last 3 deciles hold the 51.8% of employment income and the 62.4% of self-employment income. It is therefore clear that these deciles will benefit more from this type of policy and that, consequently, the inequality will increase.

6. CONCLUSIONS

Personal income distribution is the result of manifold factors of different nature, related to the functioning of every market system. Among these factors the ownership of production factors by the households and the technologies, which corresponds to a given productive structure, may be considered the most important. Personal income distribution and its inequality are linked to the shares of capital and labour owned by each individual and how they are used in the productive processes. It is a matter of fact that inequality in the primary (market) income distribution can be reduced by alternative *ex-post* equalization policies operating through transfers or through taxation. The final impact on the income of the different groups of households must be evaluated separately from the amount of the ''initial'' exogenous injection. The increase of incomes of the group that firstly benefited from the transfer is always different from the initial injection because of the interplaying of the market actors (household and companies). A suitable framework, based on the ''accounting'' multiplier approach, can assess this result.

The numerical examples, referred to the Italian economic system, allow to quantify how much an initial exogenous injection in the income of one or more endogenous accounts translates into different amounts of disposable income for all deciles and, then, in a change of the household income distribution and of its inequality. The multiplier matrix $\mathbf{M}_{33}$ (Table 9) express the inequality in the households' income distribution that, in our numerical example for the Italian economic system, can be considered as a ''structural'' feature. One meaningful result is that an increase in disposable income of the first two decile, and even more of the first four, benefits all households and in particular the richest ones. The resulting inequality level will be different depending on the endogenous accounts that benefit from the exogenous injection (Table 11). The sources of inequality in the markets of the industrialized countries are today so many and multifaceted that traditional ex-post redistributive and fiscal policies cannot alone go to the root of the problem: taxation and redistribution must be considered only tools, among many others, to reduce inequality and therefore to favour social mobility. Therefore the endogenous, structural inequality can be better highlighted with a multiplier approach rather that with the traditional ''synthetic'' inequality indices alone.

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APPENDIX

Formulae of the three inequality indices chosen for measuring inequality, based on a distribution of households classified in 10 groups (deciles of population) according to their level of disposable (net) income.

GINI INDEX *G(Y)*

$n = 10$ Population groups (deciles);
$i = 1, 2, .., n$ Population decile *i-th;*
$y_i =$ Income of decile *i-th*
$Y = \sum_{i=1}^{n} y_i$ Total income;
$p_i = 0.10$ Population's share of each decile *i-th*;
$q_i = y_i/Y$ Income's share of decile *i-th*;

$$p_i = q_i \ \forall i$$

$Q_1 = q_1 \quad Q_i = q_i + Q_{i-1} \quad i = 2, 3, .., n$ Cumulative income's share up to decile *i-th* $Q_n = 1$;
$P_1 = p_1 \quad P_i = p_i + P_{i-1} \quad i = 2, 3, .., n$ Cumulative population's share up to decile *i-th* $P_n = 1$*;*

$$G(Y) = \sum_{i=1}^{n-1} (P_i - Q_i) / \sum_{i=1}^{n-1} P_i$$
$$0 = G(Y) = 1$$

ZENGA INDEX *I(Y)*

$n = 10$ Population groups (deciles);
$i = 1, 2, .., n$ Population decile *i-th;*
$n_i = 0.10$ population's share of each decile *i-th*;
$N = 1$ Total population's share
$N_i = \sum_{j=1}^{i} n_j$ Cumulative population's share up to decile *i-th*
$y_i =$ Income of decile *i-th*
$Y_i = \sum_{j=1}^{i} y_j$ Cumulative income up to decile *i-th*
$Y = \sum_{i=1}^{n} y_i = Y_n$ Total income;
$M = Y/N$ Total mean income
$M_i^- = Y_i/N_i$ Lower mean up to decile *i-th*
$M_i^+ = \frac{Y - Y_i}{N - N_i} \ i = 1, 2, .., n-1$ and $M_i^+ = Y_n \ i = n$ Upper mean up to decile *i-th*
$I_i = 1 - M_i^-/M_i^+$ Relative variation of the lower average compared to the higher one.
I_i are punctual inequality index

$$I(Y) = \sum_{i=1}^{n} I_i/n$$

The index *I*(*Y*) has been proposed by Zenga (2007).

PALMA INDEX *IP*(*Y*)

$n = 10$ Population groups (deciles);
$i = 1, 2, .., n$ Population decile *i-th;*
$y_i =$ Income of decile *i-th*;

$$IP(Y) = y_{10}/(y_1 + y_2 + y_3 + y_4)$$

Palma is a particular specification within a family of inequality measures known as 'inter-decile ratios', of which the most commonly used is possibly the ratio between the income of the bottom 20% and that of the top 20% or its inverse. The Palma is the ratio of national income shares of the top 10% of households to the bottom 40%. It reflects Gabriel Palma's observation of the stability of the ''middle'' 50 percent share of income across countries so that distribution is largely a question of the tails (Cobham and Sumner, 2013).